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(Communicated at the meeting of September 29, 1923).

It does not unfrequently happen that in milch-cows acetonemia reveals itself a few days after parturition. Then the animals become extremely emaciated within a few days; the milk-yield decreases considerably; they give off a smell of acetone and their appetite is largely diminished. As a rule they recover after a short time, and very soon when put out to grass. The examination of the urine, the blood and the milk of more than twenty milch-cows suffering from typical acetonemia showed us that the urine of these animals often contained from 10 to 13 grms of acetone-bodies per liter. In many cases the blood contained 0.6—1 grm of these substances per liter, while the content in the milk was about half the amount in the blood. These results point to an abnormal fat-metabolism, for the acetone-bodies result mainly from abnormal metabolism of the fats [1]), the alkali-reserve of the blood was in serious cases lowered to $^1/_4$ or $^1/_3$ of the normal value. The determination of the glucose-content of the blood shows that hyperglycemia was absent. Sugar was never found in the urine. So the sugar-metabolism was in no way abnormal. The acidosis, brought about by the acetone-bodies, caused a rise of the calcium- and the ammonia-content of the urine. The disturbed fat-metabolism, was not attended with lipemia. The total content of lipoids and of fat in the blood was not or little higher than normal. This rise was chiefly due to hypercholesterolemia. Instead of about 0,1 % we found namely about 0,2 % of cholesterol in the blood-plasma. The lipoid-phosphoric acid did not seem to have increased.

Basing ourselves on the formula that expresses the border-value

[1]) GEELMUYDEN's hypothesis (Erg. d. Physiol. 1923), that acetone-bodies are normal intermediate products from the conversion of fat into sugar, may be considered highly debatable.

for the relation between ketogenic and antiketogenic substances [1]) (SCHAFFER, HUBBARD and WRIGHT) we are in a position to calculate from the obtained data (from which is also deducted that the animals consumed about 375 grams of protein) that a cow must metabolize about 1 k.g. of fat before this border-value is reached. With an ordinary diet normal cows oxidize only little fat. The above relation is then far above the border value. If the animals, as was often the case in our experiments, secrete about 120 grms of acetone-bodies a day, more than a kilogram of fat must be metabolized. So while the animals then ingest little fat with their food, about one kilogram of body-fat is burnt daily. It is evident, therefore, that in the case of acetonemia one of the organs concerned in the fat-metabolism must be seriously interfered with in its function.

The simplest way to account for this is to consider the liver as the etiological factor, as in experiments with ECK's fistula and with the reversed ECK's fistula acetone-bodies are formed in the liver. [2])

This view is supported by different observations on the diminished activity of the liver during pregnancy (N.B. acetonemia in cows manifests itself a few days after parturition) and on the abundance of fat in the liver of cows shortly before parturition.

That the disturbance regards only the function, is proved by the speedy recovery when the animals are sent to grass.

It may also be conceived that abnormally large mobilization of fat is the primary anomaly which is controlled from another organ than the liver.

That milch-cows do not easily secrete such large quantities of acetonebodies as were found with acetonemia, was evident e.g. from our experiments with cows that we allowed to fast after some injections of phloridzin (which engendered glucosuria). Indeed, some acetone occurred in the urine but only little.

Neither were the quantities of acetone-bodies considerable in the urine of cows that, on account of indigestion or for some other reason (foot- and mouth-disease) ingest little or no food.

In a diabetic cow we found the same. Although the urine contained for a considerable time from 3 to 4 % of glucose, the amount of acetone-bodies was normal or scarcely higher.

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[1]) Recent researches have shown that the border value for the healthy organism may also be taken for the organism with disturbed metabolism.

[2]) Of course these experiments do not prove that in no other parts of the organism acetone-bodies may be formed. There is this against them that their conclusiveness is greatly diminished owing to the radical measures taken, and consequently to highly abnormal circumstances.

From the wide ratio between the intake of carbohydrate and that of fat in normal cows it is clear that in milch-cows secretion of acetone takes place only with a very abnormal metabolism.

Our researches go to show that in milch-cows suffering from acetonemia waste of body fat takes place on a large scale, often about 1 kilogram daily. Lipemia, glucosuria and hyperglycemia do not occur. The total quantity of acetone-bodies amounts to about 120 grms. per day. The cholesterol-content of the blood is 50 to 100% higher than normal sometimes even more. The alkali-reserve has decreased. It is probable that the disturbed fat-metabolism is caused by intoxication of the liver.

From the Chemical Laboratory of the Veterinary University at Utrecht.

Palaeo-botany. — *"Etapteris Bertrandi Scott, a new Etapteris from the Upper Carboniferous (Lower Coal-Measures) from England, and its bearing to stelar-morphological questions."* By O. POSTHUMUS. (Communicated by Prof. J. W. MOLL.)

(Communicated at the meeting of October 27, 1923).

Remains of this plant have been found in a coal-ball from Shore, Lancashire; only the petiole is known, of which a series of transverse sections has been cut by J. LOMAX. Of this series 3 sections are present in the Palaeo-botanical collection of the Mineralogical-Geological Institute of the Groningen University (N°. 140—142); besides I have seen 6 other sections in the collection of Dr. SCOTT in the British Museum (Natural History) in London (N°. 2835—2840). The species has been mentioned by Dr. SCOTT in his catalogue of the collection as Etapteris Bertrandi, and is distinguished, as he remarks, from the other species of the genus by the well developed sinus in the xylem of the vascular bundle of the petiole.

The sections in the Groningen collection, though less in number, show some features which are not present in the British Museum specimens, and enable us to form an opinion of the relation of the species to its nearest allies.

The following description is chiefly derived from the sections present in the Groningen collection.

The order of the sections is 140—141—142; I cannot give with certainty the exact place in the series of the British Museum sections, but of the series the end is in Groningen. They are all transverse sections of the petiole, which is about $2^1/_2$ mm. thick.[1])

The epidermis is wanting; it could not be made out whether assimilating tissue with intercellular spaces had existed under the epidermis, but it is unlikely from analogy with allied species. Under these missing layers we find sclerotic tissue: thick-walled cells with a narrow lumen without intercellular spaces. In its innermost part the thickness of the cell-walls decreases and the lumen is wider. The inner cortex consists of thin-walled parenchymatous tissue without intercellular spaces; it is only preserved at the extremities

[1]) The other dimensions are shown in the microphotographs which are enlarged 45 times.

of the vascular bundle near the pinna-bar; it contains scattered cells, slightly larger than the others and with a black content. In the space caused by the destruction of the inner cortex, the pigment derived from these cells, is also scattered.

The tissue surrounding the vascular bundle has been partially preserved with it. It is thin-walled without intercellular spaces; its elements, though often very indistinct, possess a narrow lumen; they are more clearly shown in some places near the pinna-bar; there the peripheral elements seem to be smaller in size than the inner ones; this tissue may be considered as phloem. It is separated from the inner cortex by a continuous double layer of tangentially elongated cells, the endodermis.

The arrangement of the xylem-tissue of the vascular-bundle in the petiole is characteristic. Its structure is in agreement with the symmetry of the petiole and its appendices. The pinnae are placed in alternating pairs, their position to the petiole is similar to that of a leaf to an erect branch: their upper side is turned towards the petiole.

A pair of pinnae is symmetrical to a plane going through the axis of the petiole and passing between the pinnae.

The vascular bundle is symmetrical to the same plane. The structure at one end of the vascular bundle will be found at a higher or lower level to be on the opposite side. This is caused by the alternation of the pairs of pinnae. It is evident by comparing analogous structures at one end with those at the other side, that the pairs of pinnae had not quite alternated, but approached the subopposite position, often also present in the fronds of existing Ferns.

In section 142 the pinna-bundles are clearly shown, passing the cortex and lying halfway between the periphery and the vascular bundle. They are surrounded by an endodermis. The xylem-tissue is nearly round, with the narrower elements (protoxylem) lying at the inner side. The outer row of trachieds seems not to be fully differentiated yet. When followed in their downward course, the two pinna-bundles fuse, thus forming the pinna-bar, a tangentially elongated reniform bundle, with two protoxylems at its inner side. This bundle is seen at different levels in section 141, 140 and 142. At a somewhat lower level it becomes more flattened, approaches the petiolar bundle and its endodermis fuses with that of the petiolar bundle. The xylem of the latter shows in transverse section the H-form, so characteristic in this genus. From a middle band, the apolar, which is slightly thickened in its middle part and consists

of relatively large elements, two arms, the antennae, are given off at each side; they are slightly recurved and prominent at the outer side at their insertion into the apolar. Thus a more or less well developed sinus is formed. The endodermis but slightly incurves on both sides of the vascular bundle.

When followed in its downward course, the pina-bar fuses with the petiolar bundle; the ends of the xylem of the pinna-bar fuse with the two prominences on both sides of the sinus (N°. 140). Thus an elliptical mass of parenchymatous, or at any rate thin-walled tissue, is enclosed. At a lower level, as seen in section 141, the pinna-bar has wholly fused with the petiolar bundle; the enclosed parenchyma has diminished in size, especially in breadth. The peripheral loop, the downwards prolongation of the pinnabar has diminished in thickness and is but a few elements thick in its middle part.

At a still lower level its continuity is interrupted; now on the surface of the rather flat xylem a deep sinus is seen, which is bordered on both sides by prominent ridges of tracheides. These become more rounded at a lower level, and the original condition is reached again.

The continuity of the peripheral loop which is formed by the fusion of the pinna-bar with the petiolar bundle occurs in 2 of the sections of the Groningen collection. It is not shown in the London specimens. But in these the well developed sinus is clearly shown; in this feature they differ much from the other species of the genus. It is on these grounds that Scott distinguishes in his Catalogue this form from the other species; it is shown here that the deeper sinus is not an independent character but caused by the fusion of the pinna-bar, when still continous, with the petiolar bundle; a feature which is aberrant from that usual in the genus.

If one tries to make a stereometrical model of this structure, the result is shown in fig. 4. In the other species of Etapteris e.g. E. Scotti Bertrand, the pinnae-bundles are also placed in pairs and fuse on their downward course in the cortex. But at a slightly lower level before their fusion with the petiolar bundle, the pinna-bar is split up, and the two bundles resulting from this division fuse independently with the vascular bundle of the petiole. An amount of parenchyma is thus never enclosed by the fusion of the petiolar bundle with the vascular tissue coming from the pinnae. That this difference with the features in E. Bertrandi is but a relative one is shown by comparing the model of the structure of E. Scotti (fig. 5) with that of the former species. Here we see the pinna-bar

fusing with the petiolar bundle. At a somewhat lower level the continuity of the peripheral loop formed by this fusion is disturbed. The interruption thus formed is limited on both sides by the downward continuation of the halfs of this peripheral loop. The xylem of the next pinna-bar fuses with the two ridges at its extremities.

In E. Scotti we see the pinna-bar approaching the petiolar bundle too. But just before its fusion with the latter it is split up in its middle part; thus two separate bundles are formed, which fuse with the petiolar bundle. We see here the same fusion with the petiolar bundle and the same interruption in the pinna-bar; but in E. Bertrandi the highest point of the interruption is below the fusion of the pinna-bar with the petiolar bundle and in E. Scotti it lies above this point.

The interruption, the height of which is different in these two species, is always limited below by the next pinna-bar. It lies above the insertion of the pinna-bar. The relative length of the interruption to the distance between two pairs of pinnae determines the condition of the transverse section. In E. Scotti the distance between two successive pairs of pinnae is but small, often the bundles of two pairs of pinnae are shown on the same side in one and the same transverse section.

Thus the structure of Etapteris Bertrandi Scott enables us to explain the features in other more complicated species of Etapteris. On the other hand it has many points in common with simpler forms, e.g. Diplolabis Römeri (Solms) Bertrand. In this plant an interruption above the insertion of the pinna-bar is present too.

If the petiolar bundle is followed here in its downward course, which Gordon's[1]) researches enable us to do, it can be shown, that the lowest pinna-bar encloses at its inner side an amount of parenchyma by the fusion of the pinna-bar with the two sides of the interruption. At a lower level the two protoxylems which are situated on both sides of the parenchyma fuse. The parenchymatous tissue diminishes in size and ends blind below.

But throughout its course to its lowest point it is in contact with the protoxylem; it seems as if the lowest part of the parenchymatous tissue follows the course of the protoxylem when penetrating into the tracheides of the metaxylem.

It is remarkable that in these plants the protoxylems are always associated with parenchyma except in the lowest part; this parenchyma, or at any rate thin-walled tissue, is situated at the adaxial

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[1]) W. T. Gordon, 1911.

side of the protoxylem. If we assume that the protoxylem was originally wholly immersed in the metaxylem, but that afterwards the development of tracheidal elements has been arrested at the inner side, except in the very lowest part, we can explain the existence of the interruption above the insertion of the pinna-bar. For when the pinna-bar approaches the petiolar bundle and fuses with it, the parenchymatous tissue at its adaxial side is enclosed. The parenchyma associated with the protoxylems of the next pinna-bar approaches in its downwards course the peripheral loop formed by the pinna-bar next above, and as the development of the procambial cells into tracheids has been arrested, a break is formed in the loop. Through this interruption the parenchyma at the inner side of the pinna-bar is connected with that enclosed by the fusion of the pinna-bar next above with the petiolar bundle. The parenchyma which is enclosed and that which lies in the sinus is formed by the fusion of the strands of parenchyma lying adaxially to the protoxylems of successive pinna-traces. These interruptions in the peripheral loop show some resemblance to the leaf-gaps in the stele of many Ferns. Here, too, parenchyma situated adaxially to the protoxylems of the leaf-trace penetrates into the xylem of the stem, either connecting the softer tissue in the interior of the stele with that without, or hollowing the xylem of the stem by the fusion of these parenchymatous formations of successive internodes. In the first case a little strand of parenchyma, ending below blindly, can be found some distance below the insertion of the leaf-trace; in the other case this funnel in the xylem is absent. The parenchyma enclosed inside the peripheral loop may be compared with the pith, formed after the second method, but the connection of the successive parenchyma-strands of successive pinna-traces is not caused by reduction in tissue which was present before (in phylogenetical sense). This structure, caused by the peculiar symmetry of the bundle, is present on both sides.

This species agrees in the form of the antennae with E. Scotti Bertr.,[1]) but differs from it by the simpler structure of the pinnae-bundles, its smaller dimensions, and the more scattered position of the idioblasts in the inner cortex. It differs from E. shorensis Bertr.[2]) by having another form of the apolar. In this species the continuity of the pinnabar is maintained for a rather long distance, but the presence of a peripheral loop has not yet been noted. A continuous

[1]) P. Bertrand, 1909, p. 140—147, 209, pl. XVI, fig. 111, 112.
[2]) P. Bertrand, 1911, p. 30—38, pl. II, fig. 23—31, 34, 35.

peripheral loop however has been found once in E. Tubicaulis Göppert sp.[1]) from Lower Carboniferous strata of Silezia, but in many other respects it is very different from the species under discussion. Perhaps E. Bertrandi may turn out to be really a portion, e.g. the highest portion of the petiole, never before observed, of some species already known, e.g. E. Scotti or E. shorensis. By its aberrant structure however it seemed to me desirable to describe this form.

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EXPLANATION OF THE PLATE.

Fig. 1—3. Etapteris Bertrandi Scott. Transverse section of the petiole; N°. 140, 141, 142 respectively.

Fig. 4. Etapteris Bertrandi Scott. Model of the xyleem tissue of the petiolar bundle (the sides of the sinus are too sharply accentuated).

Fig. 5. Etapteris Scotti Bertrand. Model of the xylem-tissue of the petiolar bundle.

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[1]) P. Bertrand, p. 206.

Groningen. *Botanical Laboratory.*

O. POSTHUMUS: "Etapteris Bertrandi Scott, a new Etapteris from the Upper Carboniferous (Lower Coal-Measures) from England, and its bearing to stelar-morphological questions".

Fig. 1.

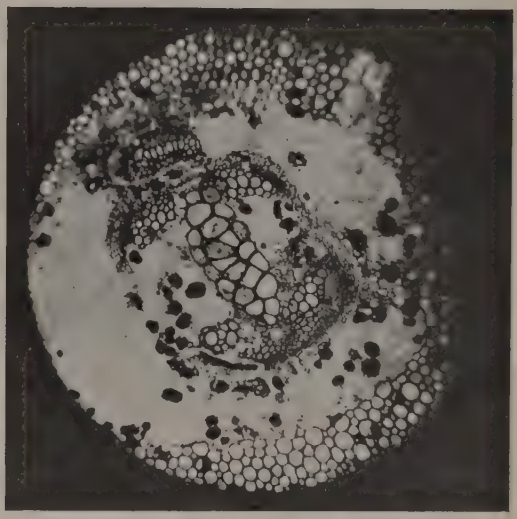

Fig. 2.

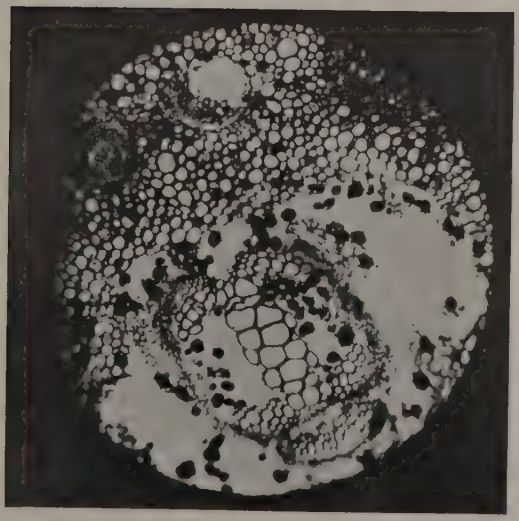

Fig. 3.

Fig. 4.

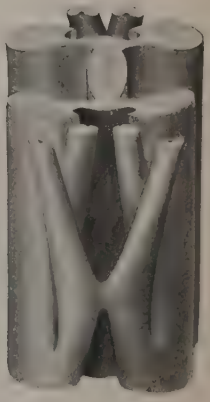

Fig. 5.

Chemistry: „*The coagulation of Hevea latex*". By Dr. O. DE VRIES. (Communicated by Prof. P. VAN ROMBURGH).

(Communicated at the meeting of January 27, 1923).

I. *Influence of the mixing-proportion of latex, water and acid, irregular series.*

It was known from previous investigations, that the coagulation of Hevea latex with acids shows irregularities. The observations of several investigators, which we intend to discuss shortly in one of the following paragraphs (§ 9), had only been made occasionally, and did not give a sufficient insight into the phenomena; therefore it seemed desirable to us to obtain a total view of the proportions, by a systematical investigation into the complete range of mixing of latex—water—acid.

§ 1. *The Latex.*

Hevea latex is a milky liquid, which, under the microscope, appears to consist of oval globules, $\frac{1}{2}$ to $2\,\mu$ in size, and showing a vivid Brownian movement; particles of less simple form occur now and then. The fact, that one has not to deal with globular particles, shows that latex is not a system liquid: liquid, an emulsion in the sense of WO. OSTWALD's classification. On the other hand, one should not speak of liquid: solid (suspension); the properties of the coagulum obtained under various circumstances, make it probable that the rubber-particles in latex have a buttery consistency, i.e. between liquid and solid. If we have to look upon this as a more or less liquid nucleus, enclosed in a more solid superficial skin, as some investigators assume, is a matter we do not intend to discuss here. If we apply FREUNDLICH's classification of the colloids to latex, then undoubtedly it is a lyophilic colloid, as shown by the hydrous voluminous gel, obtained on coagulation, and by the behaviour of the latex with regard to dehydrating and salting-out substances; on the other hand, the hydrating power of the rubber-globules is decidedly only limited, and the latex, as regards its behaviour towards mono- bi- and trivalent anions, is strongly reminiscent of lyophobic colloids. So in this classification as well, latex

occupies an intermediate place. Moreover, the rather complicated properties of the latex may be understood, if we bear in mind, that it is a vegetable juice, in which besides the rubber-carbohydrates, also proteins, resins and other colloids play a part, and in which each in its turn may come to the front.

The composition of Hevea latex is not constant. The quantity of rubber and the quantity of secondary constituents depend on several factors, which cause changes in the physiological condition of the tree; moreover the tapping-system has a great influence. Besides we have to bear in mind, that after tapping the acidety of latex principally by bacteriological transformations, increases, even to such an extent, that after twelve hours "spontaneous" coagulation sets in.

If, however, circumstances are carefully chosen, it is an easy matter, to get a regular daily supply of latex of a certain composition. For that purpose one has to be restricted to a certain group of trees, from which, according to a certain tapping-system, latex is gathered daily, which moreover is always treated in the same way. The only remaining changeable factor, the meteorological circumstances, are then immaterial, if one keeps separate the latex of those days, on which in the morning the trunks were still wet, after nocturnal rains, or on which the latex gets drenched by an early shower.

We could, by taking these precautions very carefully, obtain quite sufficiently constant results, in the coagulation-experiments to be described here, with series of observations covering several weeks.

If, however, later on, one reverts to such observations with latex of a different group of trees, or a different tapping-system, the quantitative data do not correspond exactly any more, though the general course of the phenomena remains the same. In § 8 we intend to give a few examples of the differences caused thereby, and also of the influence of the gradually increasing acidity of the latex.

The results to be discussed here, have therefore to be interpreted in such a way, that the principal features of the view are generally available, but that the limits of the different ranges may be moved more or less, according to the composition of the latex with which one operates.

Against this drawback, that one operates with a non-constant, and not arbitrarily reproducible material, we find, as a great advantage, the fact, that Hevea latex is mixable with water in any proportion. So one may easily prepare all percentages of rubber from the original percentage (30—40 %) down to the lowest one, and one may, without great difficulty, traverse and search systematic-

ally in all directions the whole range of the mixing-proportions, by serial determinations with decreasing quantities of more or less diluted latex, and increasing quantities of acid, either diluted or not. The „irregular series" being only found with the lower percentages of rubber, it was possible to determine completely the range where these occur. In most cases, described in literature, the „irregular series" have only be examined with one single or with a few concentrations, the higher or lower concentrations of the colloid not being accessible. The latex, used for most of the observations to be described here, originated from a group of trees, fifteen years old, in the experiment garden at the opposite side of the Tjiliwoeng at Buitenzorg. The trees were tapped daily, with two cuts over $^1/_4$ of the circumference of the trunk, and the latex was used for examination between 10 a. m. and noon. The percentage of rubber (on coagulation) varied from 31,0 to 32,8, and on the average amounted to 31,8 %; the acidity was 0,02—0.04 N. (cf. § 8), the acids present are principally carbonic acid, lactic acid and a little butyric acid [1]).

In 1922 complementary observations were made with latex from a few groups of trees in the Botanical garden.

§ 2. *The phenomena of coagulation.*

With the proportions, as they are chosen in the practice of the preparation of rubber, the coagulation of Hevea latex proceeds slowly. After a quarter of an hour the liquid has become thick, with the consistency of porridge; gradually it begins to cohere, and after one hour a coherent lump is formed, but still with milky serum; only after a few hours the separation into a solid coagulum and a clear serum, is complete. In other cases one causes the coagulation to proceed more rapidly, by adding more acid, so that, after one hour, one obtains a coagulum sustable for working purposes. Or one saves acid, so that only after a few hours the first phenomena occur, and the coagulum can only be worked up next morning. Sometimes the latex is used undiluted, but mostly one dilutes with water to a rubber percentage of 20 or 15 %, on account of which the coagulum becomes softer, and may be worked up more easily. The more the latex is diluted, the softer the coagulum becomes, and the stronger the contraction after the coagula-

[1]) For the composition of Hevea latex in general we may refer to „Estate Rubber, its preparation, properties and testing" by Dr. O. de Vries (Ruygrok & Co., 1920), chapter 1 and 2.

tion will be, so that more serum is set free. Only with very strongly diluted latices a flocky coagulum is separated, which does not form a coherent lump, or only gradually coheres after one or more days. If we use less acid, the coagulation sets in slowly; but with decreasing quantity of acid the spontaneous coagulation, caused by bacteria which decompose the sugars and the proteins under formation of acid, begins to play a more and more important part. Ordinary, non-sterilised latex always coagulates, even without any addition of acid, during the first night after tapping; the coagulum is then spongy by the formation of gases, and the surface exposed to the air is covered with a yellow, evil smelling layer of porridge-like separated rubber, mixed with decomposition products of proteins. So in the range of very little acid there are no mixtures, which remain liquid in the long run; the observation "liquid" may be made after a quarter of an hour or after two hours, but after 24 hours one will find the mixture coagulated. The liquid mixtures with more acid, so in what one might call the second liquid range, remain liquid for an unlimited space of time. Sometimes, after being left to themselves for several days, a separation of very thin little flocks, lying on an almost clear or whitish serum, sets in, but in any case one can control and confirm the observation "liquid" after 24 hours. This liquid range passes into the ranges of coagulation by a strip of transition, being broad especially towards the side of the higher acid concentrations, and distinctly showing different stages. The first beginning of coagulation phenomena is the appearance of a thin skin at the surface of the liquid, caused by evaporation in the air, which, on stirring with a glass rod, attaches itself to it as a streak or rolls itself up.

On approaching the range of coagulation a little more, this streak becomes thicker and more cloddy. Advancing further, we get to clotting or curdling of a greater part of the latex; a pap and finally a coherent coagulum is formed. If it is left longer to itself, the coagulation in this range of transition proceeds further; what after two hours was a pap, may after 24 hours have become a coagulum and a mixture which after two hours only showed a thick streak, has changed the next morning into a pap, or even may be coagulated. What is liquid in the middle of the second range, remains liquid even after days, but "liquid" on the limit of "streaky", may have changed into streaky after 24 hours. "Coagulated" of course remains such after one or several days, only the coagulum gradually contracts itself a little and becomes harder.

It may be clear, that with these gradual transitions, we shall

never be able to fix any sharp limits for the different ranges. The ordinary discrimination, by gently shaking or stirring, can only be a rough one. We examined if sharper criteria might be found by means of the microscope, but it appeared, that the formation of little lumps of a few or a great many small rubber-globules also took place quite gradually, without sharp transitions, and that neither the decrease nor the stopping of Brownian movement opened the way for any sharp limitation.

So most of our serial experiments were confined to judging at sight, by means of a stirring rod, only completed occasionally by microscopic oberservations. A short time, about 15 minutes, after the addition of the acid the first observation was made, which in certain ranges is already sufficient. The principal observation followed two hours after the mixture was made, and was controlled the next morning, viz. if then a stage was reached so much further advanced as might be expected from the condition, such as it was two hours after the addition of the acid. In order to be able to sufficiently overlook the whole, we have, in the following paragraphs, interpreted the observations in a somewhat simplified way; therefore, with the classifications "streaks", "curdled", "porridge", and "coagulation" we have to associate the meaning of conditions of separation gradually passing into each other, as described above.

As a rule we worked with 50 cc. of liquid for each determination, the liquid being left open to the air in a small cylindric glass till the next morning, for the last control-observation. With very small quantities of acid the mixture of latex and water was measured with a measuring-cylinder and the acid was added by means of a burette. It was not necessary to measure the diluted latex more exactly than within $\frac{1}{2}$ or 1 c.c., but the acid had to be measured exactly within one drop, especially with the very diluted latices, where the range of coagulation is narrow and sharply limited. With mixtures with larger quantities of acid, the latex, either diluted or not, was always mixed with the diluted or undiluted acid in such quantities that the total volume was 50 c.c.; while the liquid, which occupied over half of the total volume, was poured out first, and the other one added to it.

Especially in the range of a large quantity of acid, or if one uses strong acid, it is necessary to stir vigorously from the beginning, so as to prevent local coagulation, which would cause enclosure of acid, not being set free any more by further stirring. By making one same final mixture, starting from latices of different dilutions

and differently strong acids, one may however control the observations in a satisfactory way.

On account of the increasing acidity of the latex itself it is not advisable to use it more than about two hours before the observations; we only did determinations between 10 a.m. and noon, but during that time one can easily prepare a few series, in total about thirty to fifty mixtures, so that in a rather short time by many hundreds of observations one can search the whole range of mixing in all directions.

Operating in small, open cylindrical glasses, causes a certain evaporation and results in the formation of a small superficial skin of coagulated rubber, which on stirring attaches itself to the stirring-rod.

Apparently this causes an undesired complication; but for distinguishing different liquid mixtures this formation of skin appeared on the other hand an advantage, because it enables us to recognize the liquids inclined to coagulate. By repeating a few series in small Erlenmeyer-flasks, closed with a cork, we have ascertained that really these skins are formed by evaporation at the surface.

§ 3. *Hydrochloric acid.*

The easiest way to summarise the phenomena at different dilutions and different quantities of acid, is to draw these in the wellknown triangle-figure. As angular points (components) we choose water, concentrated hydrochloric acid (9.14 N) and undilated latex, i.e. a liquid with 31.8 % coagulable rubber, about 35 % totally solid substances and about 65 % water, and with an acidity of about 0.03 N. A recalculation of the results, so as to express these as quantity of acid, resp. rubber compared to the whole liquid (water of dilution plus serum) can never be correct by the phenomena of adsorption and, as regards rubber, there is not much sense in it, as coagulable rubber is a substance containing so many secondary substances in small quantities.

In the annexed figure 1 the lines show how the different mixtures are formed by mingling latex and hydrochloric acid, of different dilutions. The mixtures, in which after two hours a well coherent coagulum was formed, are marked with a little cross. As we see this range almost occupies the whole triangle; only in a narrow strip along the latex-water side, we find mixtures, which are represented by an encircled point (pap or curdling) or by a little circle (liquid), and there we can, though indistinctly on account of

the scale-size used, recognize irregular series liquid: coagulation: liquid: coagulation. This narrow strip, the range of small quantities

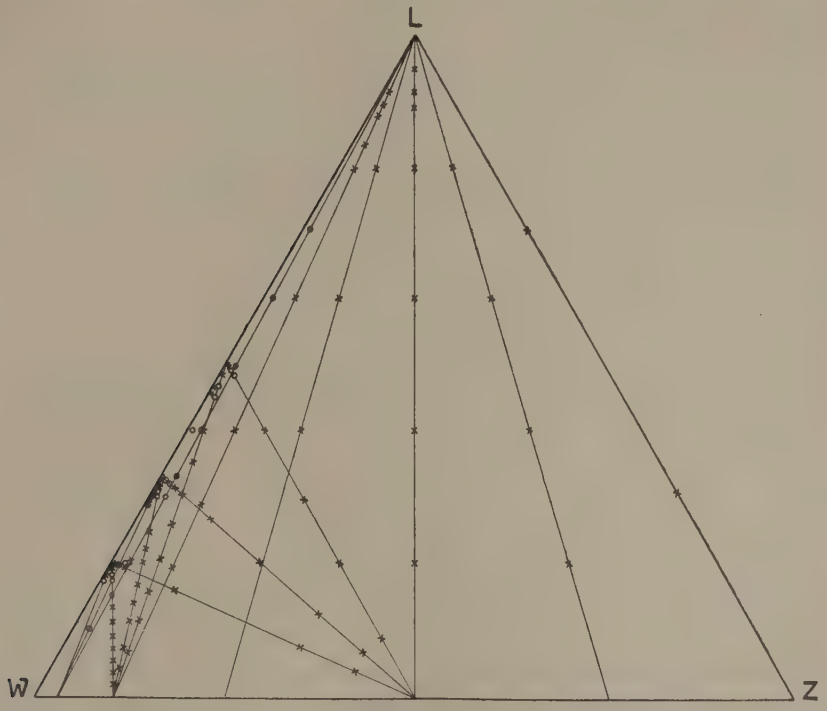

Fig. 1.

of acid, is, with hydrochloric acid, the only interesting item; the remainder of the triangle shows nothing particular, the less water the mixture contains, the harder the coagulum, while in mixtures with little water and much hydrochloric acid the serum assumes a violet tint.

The narrow strip along the latex-water side is represented on a larger scale in fig. 2, where the acid is drawn perpendicularly, as ordinate, and expressed in normality (grammolecules HCl per Liter final mixture).

For quite small concentrations of acid, at any dilution, we first come into the liquid strip, where coagulation has not yet started after two hours. After 24 hours this part shows spontaneous coagulation. At higher acid-concentration (from about 0.007 N) we find after two hours more or less strong curdling or formation of pap, and after 24 hours coagulation. The limit at which after two hours complete coagulation with a clear serum has taken place, is,

found with mixtures beyond 50% latex, to be fairly constant at 0.012 N. We should bear in mind, that this means the acidity of

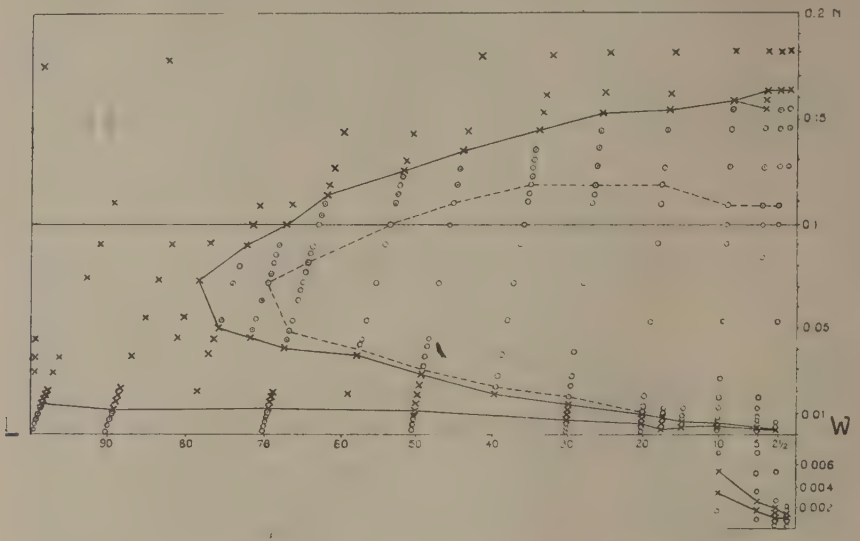

Fig. 2.

the hydrochloric acid added, which has to be increased with the original acidity of the latex, recalculated on the final mixture. For mixtures, containing less than 50% latex, this bottom-limit of coagulation is regularly lower. Because of reasons mentioned above, the observations could not be made so sharply that the relation between rubber-concentration and limit of acidity appeared quite clearly, but especially with the lower concentrations the small irregularities may be considered to be due to observation errors, and we may assume that the lowering of this limit is inversely proportional to the latex concentration.

With mixtures containing over 80% latex to which more acid is added we always get a strong coagulum, and so from the beginning we are in the range of coagulation, which fairly occupies the whole triangle of Fig. 1. At 75% latex we get the first indications that another phenomenon is about to appear, because the coagulum at first is hard, with more acid (about 0.05 N) soft or even like pap, and only with a still larger quantity of acid hard again. A distinctly liquid range only appears with mixtures with 65% latex and less.

The strip of coagulation between both liquid ranges, the lower range of coagulation, regularly decreases in latitude at lower latex-concentrations, but still remains distinctly perceptible even at the lowest concentrations (1% latex). In those very diluted liquids the

rubber does not separate itself as a coherent coagulum, but in the form of white flocks. The separation goes much quicker than with higher concentrations and with the liquids with 1 and 2½ % latex, reminds one of a titration of warm nitrate of silver with hydrochloric acid.

At those low concentrations the range of coagulation is so narrow, that, in an acidified but still unchanged liquid, one can see, with a single drop of diluted acid, the white flocks separating themselves, and that one sees the original milky liquid remain unchanged on addition of a few drops more. With mixtures with 5% latex one may get at first, with a small quantity of acid, a flocky separation, cohering fairly quickly as a coagulum; on addition of a little more acid a very soft coagulum may be formed at once. Mixtures with 2½% and 1% latex cause flocky separations, which may remain unchanged for a long time, and with which the coherence as a coagulum is the more difficult, according as the mixture contains less latex.

At higher concentrations, just above 5%, sometimes the liquid separates itself in a remarkable quick way into a coagulum and a clear serum, but the instantaneous coagulation is not found there any more. At still higher concentrations the separation of the coagulum goes slower.

The lower range of coagulation, described here, is limited by a transition to spontaneous coagulation, as discussed above; at the upper part we find a narrow range of transition, where the mixture after two hours is like pap or curdling (after 24 hours mostly coagulated). Only towards the higher latex-concentrations this strip gradually becomes a little broader, and at about 65% latex bends itself in an upper direction, limiting the top of the liquid range, and converging into the broad strip, which separates the liquid range from the upper range of coagulation. Thus the liquid range is perfectly limited, both at the upper- and at the lower side, at least till the lowest concentration, which was examined (1% latex, so 0.3% rubber in the mixture). Whether, at still smaller concentrations, the lower strip of coagulation is continued, or if both the liquid ranges meet there, has not been examined yet. The limits of the various ranges are found at the following normalities of the added acid in the final mixture: (See Table following page).

These figures are illustrated by fig. 2.

We shall now give a short description of the course of the phenomena at a few typical concentrations. To the latex-water-mixture 10% hydrochloric acid (0.914 N) was added from a burette: the

TABLE I.

Latex in the mixture.	Lower limit of coagulation.	Upper limit first range of coagulation.	Upper limit liquid range.	Lower limit second range of coagulation.
65 %	0.012	0.04	0.08	0.10
50 %	0.011	0.029	0.10	0.13
40 %	0.009	0.019	0.11	0.14
30 %	0.007	0.013	0.12	0.15
20 %	0.005	0.009	0.12	0.155
10 %	0.0035	0.0055	0.11	0.16
5 %	0.0018	0.0027	0.11	0.155
2½ %	0.0009	0.0018	0.11	0.16
1 %	0.0008	0.0011	0.14 ?	0.16

quantities were chosen in such a way that the final mixture was always 50 cc., so that the latex-concentration, at larger quantities of hydrochloric acid, decreased a little, and that the serial determinations in fig. 2 are found on slanting lines.

With a mixture with 70% latex the result of the examination two hours after the addition of acid was (cf. fig. 2):

After being left to itself for three hours, the coagulation of course had proceeded further; now $2^1/_2$ had become a pap, $2^3/_4$ a thick liquid with a good many skins, $3—4^1/_4$ remained liquid, $5^3/_4$ was softly coagulated. The mixtures in the strips of transition show a further advanced coagulation, but the true liquid mixtures remain liquid, even after 24 hours. When it is left in open small cylindric glasses, a skin is formed at the surface, evidently by evaporation, for in closed Erlenmeijer-flasks it was not formed. So the limits of the ranges are somewhat displaced, according to the moment of observation being delayed, but the phenomenon coagulated — liquid - coagulated remains. It strikes us, that the transition at the lower side of the liquid range is very acute; at the upper side however much more gradual. The little skins formed on stirring, are partly due to evaporation at the surface, or to latex, drying upon the side of the glass; yet these skins point to a higher inclination for coagulation, as such mixtures after 3 or 24 hours are coagulated further than the purely liquid ones.

c c. 10 % HCl per 50 c c mixture.	DESCRIPTION.
0.1	liquid.
0.3	liquid.
0.4	thick pap; beginning of strip of transition.
0.5	thick liquid, a few little lumps.
0.6	the same.
0.7	a somewhat thick pap, coagulating on stirring; beginning of the range of coagulation.
0.8 and 0.9	strong coagulum, serum whity.
1	coagulated, serum fairly clear, (acid added 0.018 N).
2	the same, serum fairly clear.
2¼	the same, serum white. Upper-limit first range of coagulation.
2½	liquid, a few small lumps on stirring. Therefore sharp transition.
2¾	liquid with some skin.
3	liquid; lower limit liquid range.
3½, 3¾, 4, 4¼	liquid; no skin.
4½	liquid on stirring some skin or streak. Upper-limit liquid range.
4¾	the same, a piece of skin (therefore irregularity).
5, 5¼	the same, more skin.
5½	the same, a fair quantity of skin.
5¾	like pap (at an other time only a fair quantity of streaks).
6	very soft pap, almost coagulated.
6¼	coagulated, but serum quite white, therefore far from complete. Lower-limit second range of coagulation.
6½	coagulated, fairly stiff, serum white.
7	the same, serum white.
8	the same, serum white. The percentage of latex in this mixture is 58.8 %.

An other example with 30 % latex: (See following page).

Of course the coagulum is always soft, because the mixtures only contain 30 % latex, i.e. about 10 % rubber.

Quite typical are the sharp transitions at the first range of coa-

cc 10 % HCl per 50 cc mixture.	DESCRIPTION.
0.1	liquid.
0.2	liquid.
0.3	liquid, somewhat thickish, small lump of coagulum.
0.4	coagulated, rather stiff, serum rather clear, lower limit first range of coagulation.
0.5	coagulated, serum clear.
0.6	coagulated, serum perfectly clear like water.
0.7	well-formed, but soft, jellied coagulum, serum nearly clear. Upper limit first range of coagulation.
1	quite liquid, only somewhat streaky, lower limit liquid range. Sharp transition.
1¼, 1½, 2, 4	quite liquid.
6	quite liquid, somewhat streaky, like 1. (later determination liquid without streak).
6¼	liquid.
6½	liquid, somewhat streaky, upper limit liquid range.
7	liquid, rather streaky.
8	for the greater part liquid, a good deal of streaky soft coagulum.
8½	soft coagulum, serum white. Lower limit second range of coagulation.
9	soft coagulum, serum white.
10	well-formed but soft coagulum, serum quite white.
11	the same the same.
12	the same serum almost clear.

gulation of very dilute latices; e.g. at 1 % latex (0.3 % rubber in the mixture), see fig. 2, lower, enlarged part.

The microscopic image of the liquid in the second liquid range, is, e.g. for a mixture with 2 % latex, as follows.

At a small acid-concentration almost all the rubber-globulus are still free from each other, and have a Brownian movement; only very few small lumps are seen, consisting of some little globules touching each other. Starting from an acid-concentration of about 0.02 N to increase somewhat, but by far the greater part of the particles are still free and in vivid Brownian movement. Only at

cc 1% HCl	DESCRIPTION
0.25	liquid, containing a few small flocks.
0.4	liquid, with a few small flocks.
0.45	after about 1/4 hour rising flocks are separative, so that after 1 hour the serum is almost clear.
0.	coagulates almost momentarily in flocks, rising to the surface in a layer, serum almost clear.
0.55	flocks are separated slowly. serum remains white.
0.6	liquid.
1.0	liquid.

about 0.11 N, i.e. at the upper-limit of the liquid range (see fig. 2), the number of small lumps increases and the Brownian movement decreases, and at 0.13 N hardly any particles move, and only very few show Brownian movement. At 0.14 N the decreaming begins, which, at 0.15 N, leads into the range of coagulation. From 0.10 to 0.15 N therefore, there is a gradual transition from "free particles with Brownian movement" into lumps, particles yet free but not moving, and decreaming. Whether perhaps the few little lumps, which are found in the second liquid range, were formed by a local excess of acid during addition, was not examined.

If we keep a liquid from the middle part of the second liquid range, e.g. 2% latex with 0.06 N. hydrochloric acid, in a high cylindric glass, no decreaming takes place within the first few weeks, but the Brownian movement gradually decreases. After two months most of the particles have joined into small lumps, a few consisting of two or three, but most of them consisting of a great number of globules, so that, after that time, only a fairly small number of free particles remain in Brownian movement; yet only part of the rubber is decreamed, and superficially the liquid is still equally white.

We regret we were unable to examine, whether in the second liquid range the negative charge, which the rubber-globules show in the original latex, had given place to a positive one, as required by the theory of „change of charge" [1]). Some experiments concerning

[1]) Cf. F. Powis, Z. Phys. Chem. 89 (1915), 105.
H. R. Kruyt, these Proceedings 17 (1914), 615, and 19 (1917), 1021.

the coagulation with different salts, will be described in a following communication.

We shall discuss in § 8 a few examples of the influence of the original acidity of the latex on the position of the limits of the ranges.

§ 4. *Nitric acid.*

We likewise made serial determinations with nitric acid and sulphuric acid, but less detailed, so that the limits of the different ranges were only roughly determined. For these experiments latex was used from a different group of trees, containing 28 % rubber. Fig. 3 gives the determinations for nitric acid. The general type is

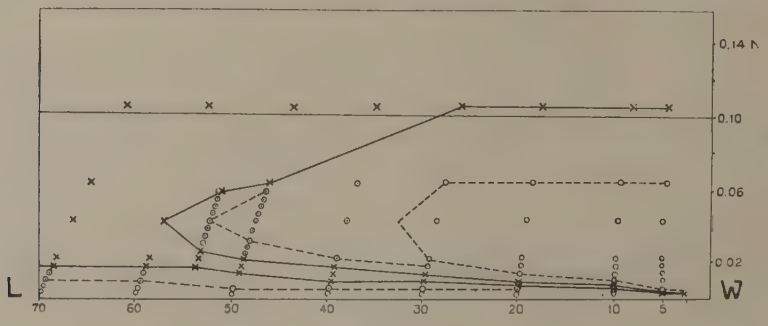

Fig. 3.

exactly the same as with hydrochloric acid, buth the liquid and the pappy ranges are smaller. Fig. 3 only goes as far as mixtures with 70 % latex; the top of the pappy range, being with hydrochloric acid at 75 % latex and about 0.07 N, is found here at a little less than 60 % latex and about 0.04 N. The top of the totally liquid range is comparatively still more displaced towards the right, so that, between both these tops, a very wide „pappy" range is found, in which we separated, by a dotted line, that part where, after two hours a thick or fairly thick pap is formed, from the part still showing fairly liquid mixtures with streaks or a beginning curdling. With nitric acid the upper-limits lie at about half the normality of that with hydrochloric acid.

In § 7 we intend to compare more closely the figures for the four acids, and also discuss more detailed the data for mixtures with 5 and 2 % latex.

§ 5. *Sulphuric acid.*

The data, which we gathered for coagulation with sulphuric

acid, have been put together in fig. 4. The large range of coagulation at acid-concentrations above 0.1 N (normal = 49 Gr. H_2SO_4 per Liter) has again been quite left out, and also the mixtures with over 70 % latex, where coagulation constantly takes place as soon as more than 0.01 N acid is added. On account of the smaller number of observations, the course of the limits in fig. 4 seems to be somewhat irregular, yet the data are sufficient to

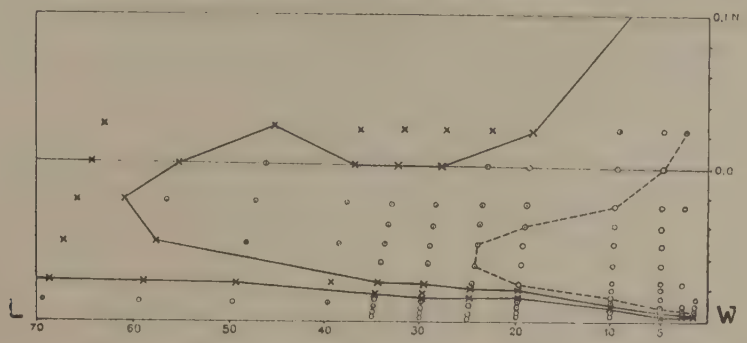

Fig. 4.

conclude, that the pappy and the liquid range, compared with hydrochloric acid and nitric acid, have shrunk still more. Figures of comparison are again found in § 7.

We may still mention, that, starting from a mixture with 70 % latex, we get a distinct indication regarding the existence of the „irregular series", though all the mixtures coagulate; the mixture with 0.04 N. acid gives a perceptibly softer coagulum than that with 0.025 or 0.05 N. acid.

§ 6. *Acetic acid.*

For acetic acid — the general and usual means of coagulation at rubber plantations — the course of the phenomena generally speaking appears to be the same as in the previous cases, but the proportions of the various ranges are quite different ones. Whilst with the three previous acids the whole range of the irregular series lies in a narrow strip along the latex-water side, which in a representation like fig. 1 is hardly discernible, the irregular series with acetic acid are extended to far higher acid concentrations, and a triangle-figure like fig. 5 opens the best general aspect. Here likewise the range of coagulation occupies by far the greater part, viz. almost $^3/_4$ of the triangle; but in the neighbourhood of the

angularpoint for water we find that over $^1/_4$ of the triangle is occupied by the liquid and pappy range, while naturally in this case also, close along the latex-water-side a first liquid range is found, not showing any coagulation on addition of a very small quantity of acid, but, after keeping, showing spontaneous coagulation by the action of bacteria.

The proper liquid range in fig. 5 is again limited by a dotted line; the pappy range is divided by a somewhat thicker dotted line into two parts, a fairly liquid and a more pappy one. Formation of a coherent coagulum takes place in the narrow strip parallel to

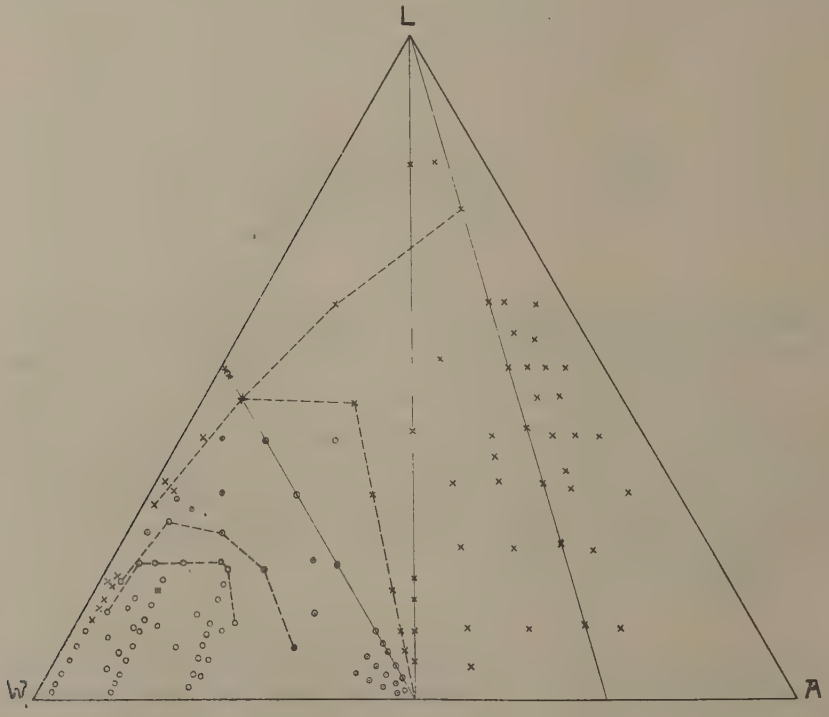

Fig. 5.

the latex — water — side and towards the side of the angulair-point Latex; the total range towards the side of the angulair-point acetic acid gives a perfect coagulation, but in the shape of flocks or as a pap, and not as a coherent lump. Both these ranges of coagulation are roughly separated in fig. 5 by a dotted line. Therefore in this respect too, there is an important difference between acetic acid and the three other acids, with which the whole range of coagulation gives a coherent coagulum.

We traced the coagulation with acetic acid once more by a considerable number of determinations, viz. in the latex of both the above-mentioned groups of trees; in fig. 5 we have represented the results, obtained with the 28 % latex of the second group (see § 4). The normality of acetic acid added is given in table 2 for the limits of the various ranges.

TABLE 2.

	Quantity of latex in the mixture in %.									
	100	80	60	50	40	30	20	10	5	2½
Limit lower liquid range.	0.004	0.003	0.003	0.003	0.003	—	0.003	0 0015	—	—
Beginning lower creamy range.	0.008	0.008	0.008	0.009	0.009	—	0.009	0.003	—	—
Lower limit range of coagulation.	0.017	0.024	0.031	0.030	0.028	—	0.016	0.006	0.0015	0.0012
Upper limit first range of coagulation.	—	—	—	0.52	0.35	0.21	0.13	0.08	0.05	0.026
Lower limit second liquid range.	—	—	—	—	—	—	0.8	0.24	0.16	0.10

On comparing these figures and fig. 5 with the results described in §§ 3—5, we distinctly see the great difference in the distance between the limits. A comparative review is given in § 7.

In judging the above figures one has to bear in mind, that the phenomena, in the sense in which we consider them here, are not exactly the same as in plantation practice. So here we take as lower limit of the range of coagulation those mixtures, where a coherent coagulum is formed after two hours, whilst with regard to the coagulation at the plantations it is moreover required, that the serum is clear or almost clear, and the coagulum sufficiently stiff to be mangled. With undiluted latex the lower limit of the range of coagulation, as it is described here (0.017 N or about 1 Gram acetic acid per Liter latex), will be lower than the amount used in practice, if we wish to mangle a few hours after the coagulation. With 50 % latex (i. e. 1 : 1 diluted) the dose (0.030 N = 1.8 Gr. acetic acid per Liter) is higher, because with diluted latex one

mangles the next day, when with a much smaller quantity of acetic acid, a coagulum fit for use has formed itself.

To this we add the results of a less complete series of observations, made in November 1922 with latex from the Botanical garden at Buitenzorg, where a few groups of trees were tapped with a cut over $^1/_4$ of the circumference.

This latex contained 37 % rubber, and had an acidity of about 0.025 N. We see that the general type is the same, that the lower limits fairly well coincide, but that, with regard to other limits, rather important differences appear, that may be attributed partly to the difference in composition and acidity of the latex, partly however, to the difference of appreciation between the observers. This example illustrates, together with the cases to be discussed in § 8, the restriction we made already in § 1, regarding the quantitative value of the results.

TABLE 3.

	Quantity of latex in the mixture in %.							
	80	60	40	20	10	5	2	1
Beginning lower creamy or pappy range.	0.009	0.010	0.009	0.006	0.0026	—	—	—
Lower limit first range of coagulation.	0.018	0.022	0.017	0.009	0.0053	0.0026	0.0020	0.0016
Upper limit first range of coagulation.	—	—	0.40	0.20	0.083	0.04	0.033	0.023
Lower limit second liquid range.	—	—	—	0.5	0.17	0.066	0.059	0.040

§ 7. *Comparison of the four acids.*

We now intend to compare amongst each other the results, obtained with the four acids. Whilst, roughly speaking the general course is exactly the same, we may notify interesting differences and conformities.

Considering first of all the top and the upper limit of the liquid range, we can use for that purpose the data mentioned in § 3—6, although they refer to two different latices, and the principal observations covered a period of over half a year, because these limits, can only be roughly determined. So we get:

TABLE 4.

	HCl	HNO_3	H_2SO_4	$C_2H_4O_2$
Top liquid range, with mixtures with latex	70%	35%	25%	25%
Top pappy range, with mixtures with latex	77%	57%	65%	57%
Upper limit liquid range for 20 % latex, at acidity	0.12 N	0.06 N	0.03 N	3—4 N
Upper limit pappy range (lower limit second range of coagulation) for 20 % latex, at acidity	0.155 N	0.10 N	0.06 N	7—8 N

The limit, at which irregular series do not appear any more — the top of the pappy range — is found for nitric acid, sulphuric acid and acetic acid at almost the same percentage of latex, but for hydrochloric acid it is somewhat higher. With all this we have to bear in mind that with nitric acid in a mixture with 60 %, with sulphuric acid in one with 70 %, a distinct interruption in the series can still be observed, owing to the coagulum, at a level of the above-mentioned top, being softer than at higher or lower concentrations of acid. A striking difference in the position of this top cannot therefore be stated with the four acids.

On the other hand there is an undeniable difference with regard to the top of the really liquid range, which, with hydrochloric acid extends to much higher latex-concentrations, than with the three remaining acids.

In the upper limit of the liquid range, i.e. the beginning of the upper curdling range, and likewise in the upper limit of this range, i.e. the lower limit of the second range of coagulation, the difference is very striking too. With acetic acid these limits are by far the highest; then follows hydrochloric acid, about halfway lower nitric acid, and half way lower again sulphuric acid. If we assume, that in the second liquid range the colloid rubber particles have changed their charge from negative into positive, the stronger coagulating action of the bivalent sulphate-ion would be fully explained; monovalent ions then would show a decided difference in the series nitrate-, chlorine-, acetate-ion.

A comparison of the action of the four acids in the first range of coagulation seemed of particular interest to us, viz. with small latex-concentrations, where, with a small increase of the acid-

concentration, we so sharply get with the three inorganic acids the phenomenon liquid-rapid coagulation-liquid, described in § 3. Therefore, for the same mixture of latex, we once more determined these limits for all four acids separately, in order to get absolutely comparable figures (which figures therefore do not fully correspond with those in §§ 3—6, as we explained in § 1 and intend to discuss more in detail still in § 8).

The figures were for the acid-concentration in normality:

TABLE 5.

	HCl	HNO_3	H_2SO_4	$C_2H_4O_2$
5 % latex, lower limit	0.0011	0.0011	0.0011	0.0015
5 % latex, upper limit	0.00265	0.00265	0.0029	—
2 % latex, lower limit	0.0007	0.0007	0.0007	0.0010
2 % latex, upper limit	0.0013	0.0013	0.0014	—

The lower limit of the range of coagulation is exactly the same with the three strong inorganic acids, and here it is quite clearly demonstrated, that, at least in this range of strongly diluted latices, the phenomenon is ruled by the positive H-ions; the action of acetic acid is somewhat weaker.

With hydrochloric acid and nitric acid the upper limit again is exactly the same; also the strips of transition (which are very narrow with these strongly diluted latices) show exactly the same phenomena if the same quantity of acid is added; so the action of hydrochloric acid and nitric acid in the lower range of coagulation is exactly the same, whilst the limit of the upper range of coagulation, as we have seen just now, is considerably lower with nitric acid. With sulphuric acid the upper limit of the first range of coagulation is a little higher; the difference is not important, but for all that, with this exclusively comparative experiment, it could be stated clearly, also because corresponding differences were noticed in the strip of transition lying above the range of coagulation. With acetic acid the upper limit is much higher (at about 0.05 and 0.026 N, see table 2) and has not been determined again in this experiment.

A determination of the hydrogen-ions concentration in these various liquids, which would be necessary for a correct interpretation of the phenomena, could not as yet take place; we only wish to draw the attention to the fact, that the subsequency of the four acids at

the upper limit of the first range of coagulation (hydrochloric acid and nitric acid — sulphuric acid — acetic acid) is not the same as at the lower limit of the second range of coagulation (sulphuric acid — nitric acid — hydrochloric acid — acetic acid).

§ 8. *Influence of the acidity of the latex itself.*

As already stated in § 1 latex is feebly acid, and on being left to itself gradually increases in acidity. The acidity of the latex, which is used for the researches, is of course not without influence on the figures obtained, though the relation need not be purely additive, as the acidity in latex is caused by carbonic acid and organic acids amongst which, after the action of bacteria, lactic acid, acetic acid and butyric acid.

First of all we made a few observations in ordinary latex and in the same latex after neutralisation with hydroxide of potassium, i.e. again for the limits, to be fixed sharply, of the first range of coagulation in mixtures with little latex. A mixture with 5% latex (percentage of rubber 1.43% needed) for the neutralisation (phenolphthalein as indicator) 16.6 cc. $^1/_{10}$ N hydroxide of potassium per Liter, and therefore was 0.00166 normal; for the original latex we calculate from these data an acidity of 0.033 N. A mixture with 2% latex (percentage of rubber 0.54%) required 6.6 cc. hydroxide of potassium and therefore was 0.00066 normal (i.e. also 0.033 N calculated for original latex).

The limits of the first range of coagulation appeared to be with hydrochloric acid:

TABLE 6.

	Own acidity	Addition hydrochloric acid in normality	
		Lower limit	Upper limit
5 % latex, original	0.00166	0.0015	0.0032
id. , neutralized	—	0.0030	0.0048
2 % latex, original	0.0066	0.0013	0.0020
id. , neutralized	—	0.00195	0.0027

We see, that the neutralization has increased the necessary addition of acid with about the amount of the own acidity of the latex. In judging the figures we should bear in mind that the neutralized

latex contains by the neutralization a small quantity of potassium salts, that may somewhat displace the limit of the ranges.

A second experiment related to the increase of the own acidity of the latex, when left to itself. The latex used for this purpose titrated, when left to itself undiluted, at 10 o'clock 0.026, at noon 0.030 and at 1.45 p.m. 0.032 N. From the observations resulted:

44½ cc. 70% latex, diluted at 10 o'clock with 5½ cc. 10% HCl (i.e. mixture 0.1 normal, belonging in the upper pappy range of transition, see Fig. 2): after one hour still liquid, but containing a fair-sized lump of streaks, and after three hours a thick pap, fairly well coagulated, with quite white serum;

the same mixture, but prepared only at 12.30 p.m. from the undiluted latex, was already coagulated, after being left to itself for one hour, though the coagulum was still very soft. So the influence of the higher own acidity of this latex was quite noticeable.

43 cc. 40% latex, prepared at 10 o'clock with 7 cc. 10% HCl (i.e. about 0.13 N, again in the middle of the upper pappy range of transition, see Fig. 2) caused after one hour a small lump of little skins, and was still liquid after three hours with a fairly strong skin;

the same mixture, prepared at 12.30 was still liquid after one hour with a small lump of skins, which was somewhat larger than in the above-mentioned mixture after one hour. So in this case the difference was noticeable, though not important.

It appears from these experiments, as might be expected, that,

TABLE 7.

	May 1920	Oct 8th 1920	Oct. 9th and 12th 1920	Oct. 14th 1920	May 1922
Own acidity undiluted latex	0.026—0.030	—	0.041—0.044	0.033	0.022
Upper limit 5 % latex	0.0027	0.0025	0.00265	0.0032	0.0044
Lower limit ib.	0.0018	0.0012	0.0011	0.0015	0.0020
Upper limit 2½ % latex	0.0018	—	—	—	—
Lower limit ib.	0.0009	—	—	—	—
Upper limit 2 % latex	—	0.0014	0.0013	0.0020	0.0026
Lower limit ib.	—	0.0007	0.0007	0.0013	0.0014
Upper limit 1 % latex	0.0011	—	—	—	0.0020
Lower limit ib.	0.0008	—	—	—	0.0014

by operating with the latex later, the quantity of acid that has to be added in order to reach a certain stage, is found to be a little smaller.

We will still give a few examples, how much the percentages of acid found may vary when latex from different origin is used, viz. for hydrochloric acid and for the limits of the first range of coagulation with mixtures with 5 and 2% latex.

If we calculate the differences in own acidity of the diluted latices, we see that the differences in acidity for the limits differ fairly strongly from them, though a general relation can be clearly noticed. In fact a strictly quantitative correspondence could not be expected as the latices differed not only in acidity but also in percentage of rubber and in secundary substances.

§ 9. *Investigations of others.*

As mentioned in the introduction, we find in literature a good many investigations, pointing to the existence of irregular series with Hevea latex.

J. PARKIN, one of the first investigators who was engaged with acid-coagulation of Hevea latex [1]), used for his experiments ten times diluted latex and stated therewith the transition liquid – coagulated — liquid. PARKIN, whose experiments were limited to small additions of acid, did not notice the second range of coagulation. As an explanation PARKIN assumed, that the protein, present in latex, is insoluble in a neutral liquid, but dissolves in alkali or acids. PARKIN was of opinion that Hevea latex is alkaline; therefore addition of acid would first cause neutralization, with precipitation of the protein and, as a result, of the rubber as well, whilst, at a higher acidity the protein would dissolve again. PARKIN further stated that with acetic acid the range of coagulation is wider than with other acids, and thought this a decided advantage for practice, because by addition of too much acid the coagulation would not fail so soon.

Because in the practice of plantations one never causes the percentage of rubber of the latex to sink below 15 or 12 % (i. e. in our terminology, one never uses mixtures with less than 50 to 40 % latex), where with acetic acid no irregular series occur, there was for a long time no further interest for these phenomena. W. CROSSLEY [2]) again gave a few figures for upper — and lower limit of the

[1]) Circulars Royal Botanic Gardens Peradeniya Vol. I (1899), 149.

[2]) India Rubber Journal 41 (1911), 1206.

first range of coagulation with a mixture with 7 % rubber (i. e. about 25 % latex) which had been preserved with formaline. We found the lower limit at 0.014 N. acetic acid, the upper limit at 0.29 N, whilst the own acidity of the diluted latex was 0.015 N. These figures correspond fairly well with ours (tables 2 and 3). CROSSLEY's lower-limit is somewhat lower and his upper-limit somewhat higher, whereby the unknown action of formaline, may have been of influence. Moreover CROSSLEY determined the lower limit of the first range of coagulation for dilutions of the above-mentioned latex with 7 % rubber, and found that, as far as a hundredfold dilution, the total acidity (acetic acid added plus calculated own acidity) decreased with great exactness proportional to the percentage of latex. For dialysed latex with a percentage of 12 % totally solid substance (i.e. a mixture with about 40 % latex) CROSSLEY [1]) found the following figures for the lower- and upper-limit of the first range of coagulation:

TABLE 8.

	Lower limit	Upper limit
Acetic acid	0.02 N	0.18 N
Trichloracetic acid	0.005	0.026
Formic acid	0.008	0.022
Hydrochloric acid	0.004	0.016
Sulfuric acid	0.005	0.018

The dialysed latex had an acidity of only 0.001 N; all the limits (except the upper-limit with sulphuric acid) are lower than those we found for normal latex, so that the dialysable serum substances in natural latex would have an anti-coagulating action.

As a criticism of these investigations B. J. EATON [2]) published a few series of observations with hydrochloric acid, nitric acid, sulphuric acid and acetic acid, which however are very incomplete and did not throw much light on the phenomena; EATON found mixtures which remained liquid, but this he attributes to a retardation of the coagulation on account of high dilution, or to an inclusion of the acid in the little lumps on partial coagulation. EATON denies the

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India Rubber Journal 42 (1911), 1345.

Bull. of the Dept. of Agric., Fed. Malay States No. 17 (1912), p. 10.

existence of a maximum-limit for the first range of coagulation, as fixed by CROSSLEY; from the above it is perfectly clear that this criticism is absolutely without ground, and that the maximum-limit, described by PARKIN and CROSSLEY does really exist; but only with mixtures with a percentage of latex below a certain limit.

G. S. WHITHBY[1]) was the first one who emphatically pointed out the existence of the second range of coagulation above the second liquid range and described a few complete series liquid coagulated — liquid — coagulated. WHITBY for these phenomena assumed the explanation that small quantities of acid have an activating influence on an enzym, which is found in latex, coagulase, which, at a small acidity, would cause the coagulation, but at a higher acidity would become inactive; the second range of coagulation then would be a direct precipitation of protein by larger quantities of acid.

We shall now compare the observations of the last two investigators with our own.

1. *Hydrochloric acid.* In Fig. 6 the limits have been taken from Fig. 2, and therein have been drawn the observations made by EATON and WHITBY.

Starting from undiluted latex EATON found with 10% acid (line 1 in fig. 6) a continual series of coagulations, but with 1 % acid

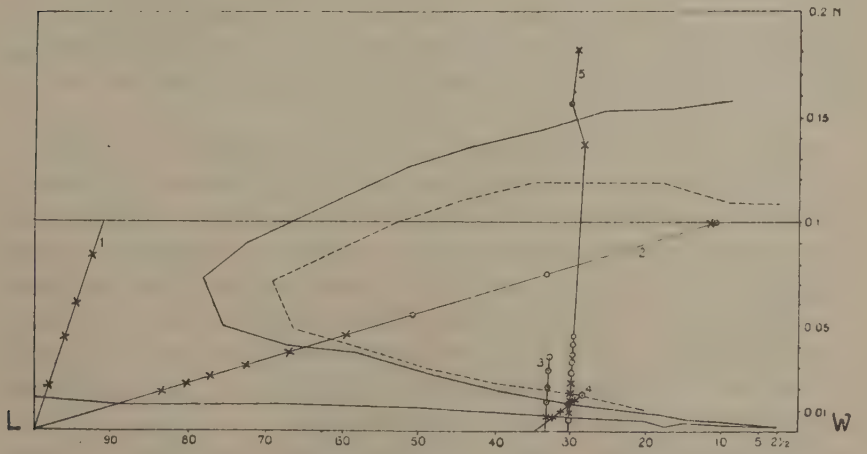

Fig. 6.

(line 2) he got into the liquid range. Two series with 1 : 2 diluted

[1]) Zeitschr. Koll. Chem. 12 (1913), 156, India Rubber Journal (London) 45 1913), 945; further Agric. Bull. of the Dept. of Agr. F.M.S. (Kuala Lumpur) 6 (1918), 381.

latex (our $33^1/_3$ %) showed him the transition from coagulated to pappy, but did not show distinctly, that he had got again into the second liquid range (lines 3 and 4). EATON did not observe the upper range of coagulation.

WHITBY made a complete series at about 30 % latex; his limits do not fully coincide with ours, which for the reasons already mentioned (own acidity latex etc.) is not astonishing, and also may be caused by wrong reproduction, as WHITBY does not mention the exact titre of his hydrochloric acid. So except small differences the observations of both investigators fit satisfactorily in the frame of our recapitulating-figure (see fig. 6 and 2).

2. *Nitric acid.* EATON made two series of observations, starting from undilated latex, and always found coagulation at increasing acidity, corresponding with Fig. 3. Moreover a series with 1 % acid with 1 : 2 diluted latex, with which he passed from the range of coagulation into a pappy range ("incomplete coagulation"), which again he attributes to the above mentioned causes (inclusion of acid in the lumps).

WHITBY also described for nitric acid a complete series, viz. for a latex with 12 % rubber (corresponding with a mixture with 40 % latex); he found at 0.016 N coagulation, at 0.021 a pap, at 0.032 and 0.052 liquid mixtures, at 0.063 a pap again, at 0.105 and 0.21 coagulation. These observations tally with ours (see Fig. 3), except both the liquid mixtures (WHITBY only says "coagulation failed to occur", which possibly may correspond with our mixtures with a little curdling).

3. *Sulphuric acid.* EATON made a series with undiluted latex, which (as might be expected) showed coagulation at all acidities; moreover one with latex diluted 1 : 3 where after the range of coagulation came a few mixtures with incomplete coagulation, and a series with latex diluted 1 : 10, where coagulated — incompletely coagulated — liquid was stated. The fact of remaining liquid is attributed again by EATON to a retardation of the coagulation with strongly diluted latex, but he does not explain in which way he accounts for the coagulated mixtures with less acid found in this series.

WHITBY only gives a short indication about a series liquid — coagulated — incompletely coagulated (pap) — coagulated, without mentioning the percentages of acid and the percentage of rubber. Probably this has been the same diluted latex with 10 % rubber (30 % latex) as in his experiments with hydrochloric acid, and therefore WHITBY probably remained at a concentration, up to which the liquid range does not reach. (cf. Fig. 4).

4. *Acetic acid.* EATON again mentions a few series with undiluted and diluted latices, in which for the diluted latices the pappy, skinny or liquid range was reached at acidities, corresponding fairly well with those found by us. For this acid WHITBY does not give any quantitative data, but only says that the first range of coagulation is much wider than with the previous acids, and that, after that, liquid mixtures are reached. With 30 % latex we did not find any liquid mixtures (top at 25 % latex), but probably WHITBY's mixture had come, by the addition of diluted acetic acid, to a lower percentage of rubber. WHITBY did not find an upper limit of the liquid range, as could not be the case (see Fig. 5) on dilution of 30 % latex with acetic acid of less than 50 %.

As we see, the data of both these investigations fit in a satisfactory way in the frame of our recapitulating-figures and their observations, partly seeming confused, are explained by the system of ranges, as they have become known to us at present.

SUMMARY.

Mixtures of Hevea Latex and water show, on addition of acids, the phenomenon of the irregular series. For hydrochloric acid, nitric acid, sulphuric acid and acetic acid the limits of the ranges (first and second liquid range, first and second range of coagulation, strips of transition) were completely fixed for all mixing-proportions of latex, water and acid (see fig. 1—5), and a comparison was made between the position of the limits for these four acids.

Buitenzorg, December 1922.

Histology. — *"On the Determination of Polarity in the Epidermal Ciliated cell. (After experiments on Amphibian Larvae)"*. By Dr. M. W. WOERDEMAN. (Communicated by Prof. L. BOLK).

(Communicated at the meeting of September 29, 1923).

It is a well-known fact that in the early stages of their life the larvae of amphibians have an epidermis, provided with ciliated cells. This cannot be observed distinctly in all species, for they differ largely as to the number of ciliated cells. Nor are these cells evenly distributed over the epidermis of one and the same larva; there are spots where they are scattered thickly, while they occur more sparsely in other spots.

The ciliary movement causes a slow rotation of the larvae while the latter are still inclosed in their jelly-like envelope. When this envelope is removed, the exposed larvae will be seen to keep up their rotatory motion owing to the ciliary movement, just as the larvae that have already left their envelope. At the same time a rather violent current may be observed in the water encircling the larva. It is self-evident that this current is strongest where most ciliated cells are collected. Strong currents are, therefore, distinguishable along certain parts of the larval body, weaker streams along other parts, which e.g. have been minutely examined by ASSHETON [1]) for Rana temporaria and Triton cristatus and have been represented in plates for larvae of various age-periods.

It appears that in these animals the first action of cilia is noticeable in larvae where the neural folds are still open, shortly before their closure. There is a strong current in the water round about the larva from head to tail along the neural walls. My own researches were made on Rana esculenta and Triton alpestris larvae. I found that in these amphibians the ciliary movement begins when the neural walls are in part united. The direction of the fluid-streams along the larval body I found to agree in the main with ASSHETON's schemata, although there were also some differences. This, however, is not to the purpose. The direction of the ciliary movement in normal larvae of Rana esculenta and Triton alpestris was,

[1]) R. ASSHETON. Quarterly Journ. of micr. Science New Series. Vol. 38. 1896, p. 465.

therefore, closely examined and represented in diagrams. It was further established that the fluid-streams flow invariably in the same direction. A reversed direction of the ciliary movement seems to have rarely been observed in metazoa. (ERHARD)[1]).

This implies such a structure of the ciliated cells that a ciliary movement is only possible in one direction, the cells present a certain asymmetry in their structure; besides their polarity (by which base and ciliated free surface are distinguished) there is an "accessory polarity" (vide ROUX[2]) for these ideas). The question has been considered whether this accessory polarity could be reversed artificially, in other words, whether the ciliated cell could by some artificial method be made to move in the opposite direction. This question is connected with another, viz. in how far the ciliary movement depends on the position of the ciliated cells relative to the axis of the body.

Experiments performed by v. BRÜCKE[3]) and those made this very year by MERTON[4]) bear on this question. They did not succeed in bringing about a reversion of the polarity. Now it has been evidenced by numerous experiments that in the embryonic development there is a period in which the ectoderm, from which the larval epidermis is derived, is still indifferent. SPEMANN[5]) e.g. found that at the beginning of the gastrulation ectoderm, destined to build up the medullary plate (so-called presumptive medullary plate), could be replaced by presumptive epidermis. Larvae developed with normal medullary plate and normal epidermis. The fate of the ectoderm-cells in that stage of development has not been, or has not yet been determined. The ectoderm is still in a high degree liable to change („umbildungsfähig"). Whether in that phase it is still completely indifferent cannot be decided without a detailed inquiry. It occurred to me that an inquiry into the polarity of the cell might afford some indication, as the polarity of the cell may already be determined before its organogenetic function. SPEMANN's experiments regard the organ-determination. Now, how about the polarity of the cell? When is it determined? The experiments in which I tried to solve these questions, I performed on larvae of Rana esculenta and of Triton alpestris in the Zoological Institute of the Freiburg University (Director Geheimrat Prof. Dr. H. SPEMANN).

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[1]) ERHARD in ABDERHALDEN's Handbuch der biologischen Arbeitsmethoden.

[2]) W. ROUX. Terminologie der Entwicklungsmechanik der Tiere und Pflanzen. Leipzig. Engelmann. 1912.

[3]) E. TH. v. BRÜCKE. Pflüger's Archiv. f. d. ges. Physiol. Bnd. 166. 1917.

[4]) H. MERTON. Pflüger's Archiv. f. d. ges. Physiol. Bnd. 198. 1923.

[5]) H. SPEMANN. Sitzungsber. d. Gesellsch. naturf. Freunde. Berlin. 1916. N°. 9.

I started by ascertaining whether there were developmental stages in which the polarity of the ciliated cell is reversible, that is stages in which the ciliated cells can be forced to move in a direction other than the normal.

After circumcision with fine glass-needles patches of ectoderm were detached from their sublayer and after a rotation of 180°—90° brought again to coalescence. After the wounds thus made were healed, which occurred in a marvellously short time, the direction of the ciliary movement was determined by examining the larvae in water in which granules of carmine had been suspended. A disadvantage of this procedure appeared to be that the borders of the wound are soon altogether invisible, so that the extent of the reversed regions cannot be traced out. For this reason I used the method adopted by W. Vogt [1]), who interchanged ectoderm patches of larvae stained vitally and those of nonstained larvae. After it had first been ascertained that vital staining with Nile-blue sulphate did not affect the ciliary action, I stained one of two larvae of the same age-period, and the other I did not. Of these two larvae fragments of ectoderm of a very well-defined shape and of the same size were excised and interchanged. In the transplantates the colour remains very well localized, it does not diffuse and enables us to recognize the contour of the implantate for many days still. Moreover, the shape of the implantate is indicative of its original position, consequently of the direction of the currents produced by the ciliary movement under normal circumstances. I shall not give an account of the various experiments, but I will describe briefly the final result of all of them.

It became evident that when a ciliated cell has once begun to vibrate it cannot be made to move in another direction. Patches of ectoderm being implanted in the wrong direction persisted to move in their original direction for days, nay, even till the ciliated cells had disappeared from the epidermis. Even before the ciliary movement has begun, its direction has already been established. When ectoderm fragments are reversed 180° before the ciliary movement begins, the ciliated cells will afterwards reveal a vibration opposite to that under normal circumstances. The youngest stages of development, however, are excepted in this respect, as it appeared that in blastulae and in incipient gastrula-stages the blastula-roof resp. ectoderm-patches can be reversed, without affecting the direction of the movement, when afterwards the larvae begin their ciliary action.

[1]) W. Vogt. Verhandl. deutsch. zoolog. Gesellsch. Bnd. 27. Sept. 1922. p. 49.

It is evident, then, that in the young stages, just referred to, the polarity of the cell has not yet been determined. It also appeared from the experiments that the determination takes place during the gastrulation. If the blastopore is still like a straight or slightly crescent-shaped slit, the future ectodermal ciliated cell is still indifferent. But as soon as the blastopore has become horseshoe-shaped and still later circular, reversion of ectoderm without reversion of the future direction of the ciliary movement is not possible.

It follows, then, that the period of determination of the polarity of the epidermal ciliated cell falls in an early stage of gastrulation.

Now we had to ask if the determination of the polarity of the cell coincided with the organ-determination.

To ascertain this we interchanged patches of presumptive epidermis and presumptive medullary plate in very young larvae, and we watched the subsequently developing ciliary movement while giving due attention to the original position (vital staining). Stated briefly the results were to the following effect: When after the operation larvae appeared with a normally developed medullary plate (part of which was consequently generated by presumptive epidermis) and with a normally developed epidermis (part of which was consequently formed by presumptive medullary plate), the larvae exhibited normal direction of ciliary movement i.e. the ciliated cells have not developed as they would have done originally, but have adapted themselves to their new environment. If the organ-determination has not yet been effected, the direction of the ciliary movement can still be influenced by the environment. But if abnormal larvae developed with a deficient medullary plate or with pieces of the medullary plate in their epidermis, then the direction of the movement appeared to have developed on the implantates according to the origin of the implantates and appeared not to have been influenced by the new environment.

Our experiments, therefore, seem to imply that the determination of the polarity of the cells and of the organogenetic function either occur synchronously or at all events with a very brief interval of time.

It should be borne in mind, however, that the organ-determination in the ectoderm does not occur everywhere at the same time. SPEMANN's and Mrs. MANGOLD-PRÖSCHHOLD's [1]) experiments have shown that this determination starts from what they have termed an „organisation centre", which is located in the dorsal lip of the blastopore.

[1]) H. SPEMANN. Arch. f. Entw. mech. der Organismen. Bnd. 48. 1921.

Fürthermore, experiments by O. MANGOLD[1]) tend to show that after the conclusion of the gastrulation, i.e. when the region of the medullary plate has already been determined, ectoderm of the ventral half of the larva can still form mesoderm or entoderm. From this we see that this ectoderm has not yet been determined.

My experiments to find an answer to the question if there is any relation between the determination of the polarity of the cell and of its organogenetic function, were carried out in the region of the future medullary plate. A more extensive investigation is required for the purpose of ascertaining whether the phenomenon that the determination of the polarity of the cell almost coincides with that of the organogenetic function of the cells holds generally or only for the region of the medullary plate.

In a subsequent communication I intend to discuss the histophysiological data regarding the ciliary movement obtained in the experiments reported in this paper.

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[1]) O. MANGOLD. Verhandl. deutsch. zoolog. Gesellsch. Bnd. 27. Sept. 1922. p. 51.

Histology. — "*A Contribution to the Histophysiology of the Ciliated Epithelium*". By Dr. M. W. WOERDEMAN. (Communicated by Prof. G. VAN RIJNBERK).

(Communicated at the meeting of September 29, 1923).

The sudden reversion of the direction of the ciliary movement which we know to be a property of a number of protozoa, is of very rare occurrence in metazoa (literature ERHARD [1]). As far as I know it has hitherto not been found in ciliated cells of amphibians.

v. BRÜCKE [2]) hit upon the idea of detaching small patches of the oral mucous membrane in frogs and allowing them to coalesce again after having turned them 180°. These experiments were hampered by all sorts of difficulties, such as inflammation, necrosis of the patches, suppuration etc. Macroscopically it could be observed in two animals that the epithelium of the patch was not destroyed, so that v. BRÜCKE was able to study the direction of their ciliary movement for 40 days. The cells continued acting in the original direction.

In three other animals the epithelium of the patch was most likely (v. BRÜCKE did not examine it microscopically) displaced by epithelium that arose from the borders of the wound. This regenerated epithelium exhibited a normal direction of the ciliary movement. Experiments made by MERTON [3]) in the past year substantiate v. BRÜCKE's data, so that it seems quite certain that in adult frogs it is not possible to reverse the direction of the ciliary movement, i.e. to alter the polarity of the ciliated cell.

Indeed, the negative results of SCHÖNE's [4]) and of WEIGEL's [5]) experiments with other epithelia had already made us suspect this; but, then, it was exactly in ciliated epithelium that the direction

[1]) ERHARD in Abderhalden's Handb. d. biol. Arbeitsmethoden.
[2]) E. TH v. BRÜCKE. Pflüger's Arch. f. d. ges. Phys. Bnd. 166. 1917.
[3]) H. MERTON. Pflüger's Arch. f. d. ges. Phys. Bnd. 198. 1923.
[4]) SCHÖNE. Die heteroplastische und homoioplastische Transplantation. Berlin 1912.
[5]) WEIGEL. Arch. f. Entw. mechan. der Organismen. Bnd. 36. 1913.

of the ciliary movement of the environment could readily be imagined to influence the movement of the cells of the turned implantate, considering our view of the conduction of the stimulus in the ciliated epithelium.

Classic experiments have in this field been carried out by VERWORN [1]). They tended to show that every ciliated cell, nay, every separate cilium has a movement of its own. However, for the regular action of the entire epithelium, in which not a single ciliated cell begins to move before its predecessor ("metachronic" ciliary movement after VERWORN), an interconnection of all those cells is indispensable. If one of the anterior vibrating elements (ciliated plates on the ribs of Beroë. Inquiry by VERWORN) is checked in its movement, all the rest will stop vibrating. If an incision is made, the part distad of the incision will not vibrate any longer with the same rhythm as the part proximad of it. The first element posterior to the incision now marks the rhythm, which is taken over by the succeeding vibrating elements.

We cannot but assume that a conduction of the stimulus must take place in the ciliated epithelium (in the free border of the cell), and that all the ciliated cells are interconnected (literature ERHARD). If this is the case, we might imagine the direction of the ciliary movement to reverse in the rotated patches of ciliated epithelium that have coalesced with the environment, since the conduction of the stimulus in these patches will now be just the reverse of the normal conduction.

But the fact that the healing of the patches of the oral mucous membrane was rather tardy and was attended with inflammation of the borders of the wound, justifies our doubt as to the existence of any normal organic connection between implantate and surroundings.

With a different object in view I have been working on larvae of Rana esculenta and of Triton alpestris, in the Zoological Institute of the Freiburg University (Director Prof. Dr. H. SPEMANN). Ectoderm patches were detached and after a rotation of 90° or 180° they were allowed to coalesce again. As the larval epidermis contains ciliated cells and exhibits a very regular ciliary movement (vide ASSHETON [2])), I was now in a position to study the effect of these rotations on the ciliary movement.

Beforehand it should be stated that the rotated patches of ectoderm in young amphibian larvae coalesce in a wonderfully short time

[1]) M. VERWORN. Pflüger's Arch. f. d. ges. Phys. Bnd. 48. 1890.

[2]) R. ASSHETON. Quarterly Journ. of microsc. Science. New Series. Vol. 38. 1896.

without reaction, so that a few hours after the operation no traces are distinguishable of the borders of the wound, even under the microscope. Now in order to verify the extent of the rotated region we had recourse to a special technique, which enabled us to recognize the contour of the rotated patch for many days together (Transplantation and vital staining after W. Vogt [1]).

The commencement of the ciliary movement in amphibian larvae nearly coincides with the closure of the neural canal. When an ectoderm region is rotated in a stage, in which the ciliary movement has just commenced or has been proceeding for some time, the ciliary movement will keep up its original direction. This lasts for days until the ciliary cells disappear from the epidermis. An influence on the rotated region by its environment cannot be made out.

If the experiment is made after the conclusion of the gastrulation, that is hours before the commencement of the ciliary movement, the result is the same. So before the movement commences its direction has already been determined.

Only in blastulae and the youngest gastrula stages can the future ciliary movement be influenced successfully.

From these experiments it may, therefore, be concluded that after the conclusion of the gastrulation the polarity of the ciliated cell has been determined. The following experiments were now made with stages immediately succeeding the conclusion of the gastrulation. I have extended the experiments to various spots that might be considered as a source of the ciliary movement. They were turned long before the movement began. Nevertheless the process of the ciliary movement in the non-rotated regions progressed quite normally.

In another set of experiments vibrating patches of ectoderm were implanted into young stages that did not yet possess ciliary movement. Now it might be supposed that on the appearance of the movement, its direction would be dictated by that of the implantate. In every experiment this influence failed to appear.

Furthermore, non-vibrating patches of ectoderm (of very young stages) were implanted into older larvae with vibrating epidermis.

Now also it might be supposed that, when the ciliary movement of the implantate commences, its direction would be determined by the epidermis of the host.

It appeared, however, that the ciliary movement of the implantate commenced simultaneously with the movement of the larva

[1] W. Vogt. Verhandl. deutsch. zool. Gesellsch. Bnd. 27. Sept. 1922. p. 49.

from which the implantate had been derived and that the direction of the movement was determined by the origin of the implantate, *not* by the new surroundings (a true case of "Selbstdifferenzierung" after Roux).

Now it may justly be assumed that in the younger stages that I operated upon, the implantates are readily taken up into organic connection with their surroundings. In experimental embryology numerous cases are known in which such an implantate behaves in every respect like the region it has displaced. Nay, the fact that the implantate is competent to incite remote cells to display their organogenetic function, points indeed to conduction of a stimulus from the implantate to its environment, which also implies that the implantate has an organic relation with its environment.

In order to account for the beautiful metachronism in the ciliary movement it is generally supposed that there is a conduction of stimuli from one ciliated cell to the other. Recent experiments by WINTREBERT [1]) have proved, moreover, that this conduction exists and takes place in young stages without the help of the nervous system, i.e. in the epithelium alone.

The experiments on blastulae and young gastrulae go to show that the turned patches vibrate co-ordinately with their environment. This implies that not only the direction of the movement of every cell is opposite to that in which the cell would originally have moved, but also that the regulation of the ciliary movement is reversed and agrees with the sequence of vibrations in the environment of the rotated patch.

This co-ordinate movement simultaneous with the environment proves: 1° that the patch is apparently stimulated by the environment (so that the conduction of stimuli has not been interrupted); 2° that the polarity of the cell is reversed; 3° that the direction of the stimulus-conduction is reversed. If in an older larva a patch of epidermis is turned, then the cilia on this patch persist in moving co-ordinately, but not in co-ordination with the environment. There is not a single reason why the patch should not receive stimuli from its environment now. Various experimental embryological data point to the fact that also in these stages such a relation arises again after the wounds have been healed. If this is the case, the results of the experiments with older stages would imply 1° that then the polarity of the cell is not reversed; 2° that the conduction of the stimuli still takes place in the original direction.

[1]) P. WINTREBERT. Comptes rendus de l'Acad. des Sciences. Paris T. 172. 1921, p. 934.

We are, therefore, impressed with the idea that the direction of the conduction and the polarity of the ciliated cells are determined simultaneously, and that conduction of the stimulus is possible only in a special direction. We are justified in assuming that this phenomenon depends on the nature of the connection between the ciliated cells. However, microscopical researches have not yet produced positive evidence of this nature.

Meteorology. — "*A non-tangent infralateral arc*". By Dr. S. W. Visser. (Communicated by Prof. E. van Everdingen Jr.).

(Communicated at the meeting of October 27, 1923).

On 24th June 1923 I saw at the Astronomical Observatory at Lembang a beautiful halo, which I will describe in the following pages.

Already early in the morning a mock-sun was visible on the right of the sun. Direct measurements of its distance were impossible, as the sun itself was hidden by thick clouds. About twelve o'clock a very bright lower tangent arc appeared, which after a few minutes became so intensely luminous as to be visible from time to time through the lower clouds. Soon this arc spread and developed into a complete circumscribed halo within which a weak ordinary ring became also visible. I succeeded between $12^h\,17^m$ and $12^h\,49^m$ in taking some 26 measurements of both rings by means of the cloud theodolite, mounted at the Observatory expressly for observations of halo's. To these measurements I will refer afterwards. In the mean time I kept a keen lookout for other halo's. Not before $12^h\,49^m$ my effort was rewarded by the apparition of a spot of light on the left below the sun, near the place where the mayor ring (46°) was to be expected. This spot soon grew more intense and developed into a short, oblique arc. Colours (red and green) were visible. On the other side of the sun nothing could be observed, because there the Cu-cloud around the Tangkoehan Prahoe shielded the Cirrus layer from our vision. I now concentrated my full attention on this arc and obtained 12 measurements until $1^h\,4^m$. Sometimes clouds prevented the observation. Moreover between $12^h\,16^m$ and $1^u\,2^m$ fourteen control-observations of the sun were made. At $1^h\,4^m$ the lower cloud had so much increased, that the measurements had to be finished. At half past seven in the evening the Ci-St proved to be still present, there was a bright lunar halo, but without any particularities.

The same halo's were seen by M.M. Voûte and Rijken Rapp

during their railway journey between Tjimahi and Bandoeng. However they saw the small arc not on the left (west) of the sun, but on the right (east). Though on the left hand side the sky presented an equally smooth Cirrus-veil as on the right, nothing was to be seen there. According to RIJKEN RAPP the arc was intensely coloured and bent like a portion of the greater ring. I have not been able to note any curvature at Lembang.

Before discussing my measurements I give here a short review of the theory of the infralateral arc.

BRAVAIS explains the arc by the refraction of light in ice-crystals with a horizontal principal axis, the light entering at a vertical basal plane (the hexagonal terminal plane of the crystal) and leaving at a sideplane of the prism. The refracting angle is 90° then. For a definite position of the principal axis (defined f. i. by its azimuth) we get a circular arc perpendicular to this axis and at a distance from the sun, depending on the sun's height. In a simple way we may imagine this circle by drawing the case of the circumzenithal arc and rotating the drawing then over 90°, so that the axis which at first was vertical, now gets a horizontal position. To each azimuth of the axis such a circle belongs. The envelope of all these minor circles is the infralateral arc. One among these circles is tangent to the greater ring. For the rest, this arc does no more then the circumzenithal arc fulfil the conditions for minimum deviation of the refracted rays of light.

PERNTER (Meteorologische Optik, 1st Edition) sticks to these conditions. He considers the arc as a "Lowitz arc of the greater ring" and deduces the form and position of the lateral arcs to the smaller and greater rings in an exactly analogous way. Without going into the details of the calculations, we may state, that the arc according to PERNTER in consequence of the conditions for minimum deviation which he imposes, generally will be less distant from the ring than BRAVAIS's arc.

BESSON (Sur la Théorie des Halos, Paris 1909, p. 51, p. 70) has shown, that PERNTER's theory is not very satisfactory. EXNER (PERNTER-EXNER, Meteor. Optik, 2nd Edition 1922 p. 405) concurs in this opinion and develops a new theory. During the normal fall of an ice-prism the principal axis and one of the bigger diagonals of the hexagon are placed horizontally. An infralateral arc may than be formed by light, entering the basal plane and emerging from one of the oblique prism-planes. The plane perpendicular to the refracting edge is inclined to the horizon at an angle of 30°. For one definite

height of the sun (27° 45') the lateral arc is tangent to the ring. For all other suns-heights the arc deviates towards the outer side. If we allow rotations about the principal axis, minimum deviations are possible up to a suns-height of 80° 50'. According te EXNER (f.i. pag. 402) measurements are lacking. However there exists one by BESSON (l.c. pag. 71). 23rd April 1908 with a suns-height of 53° he saw an infralateral arc on the left below the sun at a height of 19°, whereas from BRAVAIS's theory a height of 18° 57' would follow.

This case bears some resemblance to that of Lembang. "Three minutes afterwards" BESSON writes "the ring of 22° and the circumscribed halo appeared, complete but scarcely visible". In both observations the same forms of halo's appear. [1])

For the measurements at Lembang as a rule the red of the arc was vised at. Once green was measured. Two times the left- and righthand ends of the red were determined.

The readings and some distances and angles calculated from these have been entered in the following table.

Nr.	M. J. T.	⊙ az.	⊙ h'	Az_w	h_w	Δ_w	A_w	h_b	Δ_b	A_b	h_{w-b}	Δ_{w-b}	A_{w-b}
1	12u 50m	N 17.2 W	58.2	N 60.1 W	22.8	46° 44'							
2	"	"	"	58.3	22.4	46 17	58° 19'	20.6	45.7	48.0	+1.9°	+0.9	10.1°
3	12 51	17.6	"	60.3	22.4	47 0							
4	"	"	"	58.9	22.5	46 17							
5	12 52	18.0	58.1	60.4	22.9	46 23	56 10	20.5	45.5	48.4	+1.4	+0.9	7.8
6	"	"	"	56.4	20.9	46 24							
7	12 54	18.8	57.9	58.2	21.5	46 10	54 57	20.5	45.3	48.8	+1.0	+0.9	6.2
8	12 55	19.3	57.8	56.4	20.3	46 11	54 10	20.5	45.2	49.1	+0.5	+1.0	5.1
9	12 56	19.7	57.7	60.3	21.8	46 19							
10	12 57	20.1	57.6	60.2	20.3	47 22	55 13	19.4	46.2	50.5	+0.4	+1.2	4.8
11	12 58	20.5	57.5	59.6	21.1	46 10	54 39	20.4	45.3	49.6	+0.7	+0.9	5.1
12	1 4	23.0	57.1	59.5	20.0	45 46	51 17	20.3	45.2	50.3	−0.3	+0.6	1.0

The observations 5 and 9 refer to the lefthand end, 6 and 8 to

[1]) See also: E. VAN EVERDINGEN. Halo's in April, Hemel en Dampkring 21, 1923, p. 216, 217.

the righthand end, 10 to the green. The time is Middle-Java time. The suns-height and azimuth were calculated and with these the readings of the theodolite were reduced. Az_w and h_w stand for the observed azimuth and height of the arc; Δ_w is the distance of the observed points from the sun calculated from the 4 foregoing columns; A_w is the angle between the suns vertical and the radiusvector from the sun to the arc, deduced from the observations.

The column under Δ_w shows, that the points measured deviate sensibly from the ring, for the red of the ring is formed at a distance of 45°6′ from the sun. The mean deviation is 1.1°. Gradually the distance decreases, but for Nr. 12 it is still 0.6° larger than that of the ring. This deviation is so big and so systematic, that it is impossible to think of observational errors. Indeed there is question here of a *non tangent* arc. The position of the tangent-point of the arc was calculated according to BRAVAIS's theory. The results have been entered under h_b, Δ_b and A_b. The calculation was carried through for the 10th observation for green ($n = 1.3115$) for the rest for red ($n = 1.307$). In taking the differences between observation and calculation the first four points, which in consequence of the initial weakness of the arc happened to be less accurate than the others, were combined to a mean value. The observations 5 and 6, 8 and 9, which refer to the ends of the arc, were substituted by their mean values.

Almost all the observed points are too high (column h_{w-b} gives the difference observation and calculation), but they approach the height calculated from theory. The angle A, which according to theory should increase for a sinking sun, in reality rapidly decreases. In consequence the difference between observation and calculation decreases from 10° tot 1°. Finally, the distance from the sun remains almost constantly 0.9° too big, hardly showing any tendency to decrease.

During the whole time of observation the arc remains outside of Bravais's arc; the position with respect to the sun approaches more and more that of the theoretical tangent point.

This arc deviates from that of BRAVAIS and hence still more from that of PERNTER. No more is it in harmony with EXNER's theory. For in this case we have to assume a normal plane inclined at an angle of 30°. In our case the rays of the sun are in their turn inclined to this plane at an angle of at least $57.1° - 30° = 27.1°$. The smallest distance from the arc to the sun is then 57.6°, which is quite out of question for the observed arc.

As was explained above, crystals showing various orientations of the principal axis in the horizontal plane contribute to the formation of the infralateral arc.

That is why I calculated what position in space the axis ought to present in order to give rise to the phenomenon as it was observed. I supposed, that the refraction took place in the normal plane — for in this case the deviation is a minimum and the intensity of light a maximum.

We consider the spherical triangle ZSN, formed by the zenith Z, the sun S and the vanishing point of the crystal-axis N. We know ZS, the complement of the suns-height, $\angle$S, the supplement of the angle A we already determined, and arc SN. The latter is the angle of incidence i of the rays of light and is to be deduced from the observed Δ. Arc ZN and $\angle$Z may then be calculated, ZN gives the height of the vanishing point, $\angle$Z is the difference in azimuth with the sun. From this follows the azimuth of the axis, as the sun's azimuth is known.

The results are as follows:

Nr	i	ZN	Z	az.ax.
1—4	74.6°	92.4°	55.2°	N 72.7°W
5—6	74.2	93.0	53.2	71.2
7	73.5	93.0	51.8	70.6
8—9	73.2	93.7	51.3	70.8
10	74.4	93.6	52.4	72.5
11	73.5	93.4	51.6	72.1
12	72.2	93.9	48.1	72.1

Hence in the mean the crystal-axis is inclined at an angle of 3°.3 to the horizon and its azimuth is N 71.8 W.

The position of the axis appears to be stationary. The differences with the mean value are as a rule below 1°. The conclusion is the more remarkable for the azimuth, as the difference in azimuth with the sun decreases more than 7° during the observations.

In trying to find an explanation of such a position by taking into account the influence of gravitation, wind [1]) and atmospheric

[1]) M. Pinkhof. Bijdrage tot de theorie der halo-verschijnselen. Verhandelingen Kon. Akademie van Wetenschappen 1e Sectie, Dl. 13, N°. 1, p. 21, 1919.

electricity on the position of the ice-crystals, I met among others with the difficulty, that the complete development of the circumscribed halo seemed at variance with the explanation proposed. Therefore I hope to come back to this point afterwards. For each explanation however the observations on the ring and its envelope may be wanted. They follow therefore as the concluding part of my remarks.

BRB = lower tangent arc; O.H = circumscribed halo; K = ring of 22°; l = left; r = right. The remaining symbols have the same meaning as in the other tables.

The mean of the 6 observations on the red of the ring is 21° 54′, only 2′ more than that found from the measurements on the top.

The calculated Δ_b is meant for white light, the observed Δ_w for red. Leaving apart the 4 very discordant differences for the first 4 observations, the mean difference observation minus calculation is —0.3°, that means exactly the difference in distance for red and white. Hence these observations of the circumscribed halo are in harmony with the calculation for red. In the 15 measurements on the ordinary ring however, on the contrary a very distinct difference of +0.3° remains.

A. Measurements of the upper and lower top (red).

M. J. T.	⊙ h	height of the top		Δ	
		lower	upper	lower	upper
12u 18m	59.7°	37.5°	—	22.2°	—
21	59.6	—	81.3°	—	21.7°
22	59.6	—	81.7	—	22.1
24	59.5	38.0	—	21.5	—
31	59.4	—	81.1	—	21.7
32	59.3	37.7	—	21.6	—
35	59.2	37.1	—	22.1	—
36	59.1	—	80.9	—	21.8
48	58.4	36.3	—	22.1	—
			mean	21.9	21.8

Mean of all measurements 21° 52′. for red according to PERNTER 21°34′

B. Measurements of the ring and the circumscribed halo.

Nr.	Time	⊙ h	⊙ az	h_w	az_w		Δ_w	A_w	Δ_b	Δ_{w-b}
1	12u 17m	59.7°	N2.8°W	38.1°	No°. oW	BRB r	22°37′	5°49′	21.9°	+0.7°
2	17	59.7	2.8	38.1	7.6	„ 1	22 48	9 55	22.0	+0.8
3	19	59.7	3.5	39 3	—11.7	„ r	22 32	31 58	22.8	—0.3
4	20	59.7	4.2	39.3	17.2	„ 1	21 59	27 42	23.0	—1.0
5	26	59.5	7.0	58.5	55.0	O.H 1	24 11	108 34	24.4	—0.2
6	27	59.5	7.4	58.5	50.6	K 1	21 53	106 20	—	—
7	29	59.4	8.3	59.4	56.0	O.H 1	24 11	110 40	24.4	—0.2
8	29	59.4	8.3	59.4	52.3	K 1	21 58	109 1	—	—
9	37	59.0	11.8	59.0	60.0	O.H 1	24 15	110 47	24.5	—0.2
10	38	59.0	12.2	59.0	—36.7	O.H r	24 35	111 5	24.5	+0.1
11	41	58.9	13.5	58.9	56.8	K 1	21 58	108 45	—	—
12	43	58.8	14.3	58.8	—28.8	K r	21 54	108 22	—	—
13	43	58.8	14.3	58.8	—33.0	O.H r	23 58	110 23	24.5	—0.5
14	46	58.5	15.6	43.7	—16.4	O.H r	24 34	69 11	25 0	—0.4
15	47	58.5	16.0	51.6	—20.8	K r	21 52	87 15	—	—
16	47	58.5	16.0	50.9	—25.7	O.H r	24 31	90 0	25.1	—0.6
17	49	58.3	16.8	41.1	44.8	K r	21 46	72 32	—	—

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Chemistry. — "*In-, mono- and divariant equilibria*". XXIV. By Prof. F. A. H. SCHREINEMAKERS.

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Components and composants.

In our considerations we have represented the composition, the thermodynamical potential etc. of the different phases with the aid of the quantities of the components; we may, however, also represent them in another way.

For example we take a quaternary system with the components X Y Z and U. The composition of an arbitrary phase may be represented by:

$$F = x\,X + y\,Y + z\,Z + (1 - x - y - z)\,U \quad . \quad . \quad . \quad (1)$$

wherein $x\,X$, $y\,Y$ etc. represent x quantities of X, y quantities of Y, etc. In a system of coördinates with the axes x y z the component U is situated, therefore, in the origin of the coordinates; we call U the fundamental-component.

We now take in the quaternary system under consideration, four arbitrary phases M N P and Q; we may represent the composition of the phase F by:

$$F = m\,M + n\,N + p\,P + (1 - m - n - p)\,Q \quad . \quad . \quad . \quad (2)$$

As definite values of m n and p belong to each composition of F, we may, therefore, also consider the composition of F as a function of m n and p.

We call the phases M, N, P and Q, in which we express the composition of a phase F, the *composants* of the system; we shall call Q the *fundamental composant*.

When we represent the composition of a phase F by (1), consequently expressed in its components, then its thermodynamical potential, its free energy etc. a function of x y and z; when we represent the composition by (2), consequently expressed in composants, then we may represent its thermodynamical potential, its free energy etc. also as functions of m n and p. Of course there exist relations between those two way of representations; we shall deduce them further.

We now consider the equilibrium between a variable (f.i. liquid)

phase L and a constant (f.i. solid) phase F. The composition of L may be x, y, z and $1-x-y-z$ expressed in the components, the composition of F: a, b, c and $1-a-b-c$.

When we deduce in some way the condition of equilibrium for this system $F+L$, then we find:

$$\zeta-(x-a)\frac{\partial\zeta}{\partial x}-(y-b)\frac{\partial\zeta}{\partial y}-(z-c)\frac{\partial\zeta}{dz}=\zeta_1 \quad . \quad . \quad . \quad (3)$$

wherein ζ represents the thermodynamical potential of L and ζ_1 that of F.

We now express the composition of L and F in the composants M, N, P and Q. Let be the composition of L: m, n, p and $1-m-n-p$; that of F: α, β, γ and $1-\alpha-\beta-\gamma$. In a similar way as we may deduce (3) we then find:

$$\zeta-(m-\alpha)\frac{\partial\zeta}{\partial m}-(n-\beta)\frac{\partial\zeta}{dn}-(p-\gamma)\frac{\partial\zeta}{\partial p}=\zeta_1 \quad . \quad . \quad . \quad (4)$$

Let us take two variable phases L and L_1 (f.i. two liquids or vapour + liquid or mixed crystals + liquid etc.). We express the composition of those phases with the aid of the components viz. $x\ y\ z$ and $x_1\ y_1\ z_1$, with the aid of the composants viz. $m\ n\ p$ and $m_1\ n_1\ p_1$. In the first case we find as conditions for equilibrium:

$$\left.\begin{aligned}&\zeta-x\frac{\partial\zeta}{dx}-y\frac{\partial\zeta}{\partial y}-z\frac{\partial\zeta}{\partial z}=\zeta_1-x_1\frac{\partial\zeta_1}{\partial x_1}-y_1\frac{\partial\zeta_1}{\partial y_1}-z_1\frac{\partial\zeta_1}{\partial z_1}\\&\frac{\partial\zeta}{\partial x}=\frac{\partial\zeta_1}{dx_1}\qquad\frac{\partial\zeta}{\partial y}=\frac{\partial\zeta_1}{\partial y_1}\qquad\frac{\partial\zeta}{\partial z}=\frac{\partial\zeta_1}{\partial z_1}\end{aligned}\right\} \quad . \quad (5)$$

When expressed in the composants, we find:

$$\left.\begin{aligned}&\zeta-m\frac{\partial\zeta}{\partial m}-n\frac{\partial\xi}{\partial n}-p\frac{\partial\zeta}{\partial p}=\zeta_1-m_1\frac{\partial\zeta_1}{\partial m_1}-n_1\frac{\partial\zeta^1}{\partial n_1}-p_1\frac{\partial\zeta_1}{\partial p_1}\\&\frac{\partial\zeta}{\partial m}=\frac{\partial\zeta_1}{\partial m_1}\qquad\frac{\partial\zeta}{\partial n}=\frac{\partial\zeta_1}{\partial n_1}\qquad\frac{\partial\zeta}{\partial p}=\frac{\partial\zeta_1}{\partial p_1}\end{aligned}\right\} \quad . \quad (6)$$

Generally we may say that the equations for equilibrium have a same form, independent on the fact whether they are expressed in components or in composants.

We now shall consider more in detail the relations between components and composants. For this we take again the composants $M\ N\ P$ and Q. We represent, expressed in components, the composition:

of M by $\alpha_1\ \beta_1\ \gamma_1$ and $1-\alpha_1-\beta_1-\gamma_1$
,, N ,, $\alpha_2\ \beta_2\ \gamma_2$,, $1-\alpha_2-\beta_2-\gamma_2$
,, P ,, $\alpha_3\ \beta_3\ \gamma_3$,, $1-\alpha_3-\beta_3-\gamma_3$
,, Q ,, $\alpha_4\ \beta_4\ \gamma_4$,, $1-\alpha_4-\beta_4-\gamma_4$

In order to express the composition of a phase

$$F = x\,X + y\,Y + z\,Z + (1 - x - y - z)\,U \quad . \quad . \quad . \quad . \quad (7)$$

in the four composants, we put:

$$F = m\,M + n\,N + p\,P + (1 - m - n - p)\,Q \quad . \quad . \quad . \quad (8)$$

so that Q is the fundamental composant. As (7) and (8) represent the same phase F, it follows:

$$\left.\begin{aligned} m(\alpha_1 - \alpha_4) + n(\alpha_2 - \alpha_4) + p(\alpha_3 - \alpha_4) &= x - \alpha_4 \\ m(\beta_1 - \beta_4) + n(\beta_2 - \beta_4) + p(\beta_3 - \beta_4) &= y - \beta_4 \\ m(\gamma_1 - \gamma_4) + n(\gamma_2 - \gamma_4) + p(\gamma_3 - \gamma_4) &= z - \gamma_4 \end{aligned}\right\} \quad . \quad (9)$$

so that $m\,n$ and p are defined.

In order to define, however, $m\,n$ and p from (9) the determinant, formed by the coefficients of m n and p may not be zero. Consequently in general we have the following:

in a system of n components we may choose n arbitrary phases like composants, notwithstanding their determinant is not zero.

For a ternary system this means: we may choose three arbitrary phases as composants notwithstanding those are not situated on a straight line. In a quaternary system we may take 4 arbitrary phases as composants notwithstanding those are not situated in a flat plane.

When we represent the composition of a phase F as in (8) with the aid of composants, then we may consider the thermodynamical potential ζ of this phase also as a function of m n and p. Hence it follows:

$$\frac{\partial \zeta}{\partial m} = \frac{\partial \zeta}{\partial x}\cdot\frac{dx}{dm} + \frac{\partial \zeta}{\partial y}\,\frac{dy}{dm} + \frac{\partial \zeta}{\partial z}\cdot\frac{dz}{dm} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and still 2 similar relations, which we obtain by substituting in (10) m by n and p. With the aid of (9) we now find:

$$\left.\begin{aligned} \frac{\partial \zeta}{\partial m} &= (\alpha_1 - \alpha_4)\frac{\partial \zeta}{\partial x} + (\beta_1 - \beta_4)\frac{\partial \zeta}{\partial y} + (\gamma_1 - \gamma_4)\frac{\partial \zeta}{\partial z} \\ \frac{\partial \zeta}{\partial n} &= (\alpha_2 - \alpha_4)\frac{\partial \zeta}{\partial x} + (\beta_2 - \beta_4)\frac{\partial \zeta}{\partial y} + (\gamma_2 - \gamma_4)\frac{\partial \zeta}{\partial z} \\ \frac{\partial \zeta}{\partial p} &= (\alpha_3 - \alpha_4)\frac{\partial \zeta}{\partial x} + (\beta_3 - \beta_4)\frac{\partial \zeta}{\partial y} + (\gamma_3 - \gamma_4)\frac{\partial \zeta}{\partial z} \end{aligned}\right\} \quad . \quad . \quad (11)$$

From those equations it follows also, with the aid of (9)

$$m\frac{\partial \zeta}{\partial m} + n\frac{\partial \zeta}{\partial n} + p\frac{\partial \zeta}{\partial p} = (x - \alpha_4)\frac{\partial \zeta}{\partial x} + (y - \beta_4)\frac{\partial \zeta}{\partial y} + (z - \gamma_4)\frac{\partial \zeta}{\partial z} \quad . \quad (12)$$

Above we have seen that for an equilibrium $F+L$ as well equation (3) as (4) is valid; we are able also to prove this by converting equation (3) into (4) with the aid of the above relations. We write (3) in the form:

$$\zeta - x\frac{\partial\zeta}{\partial x} - y\frac{\partial\zeta}{\partial y} - z\frac{\partial\zeta}{\partial z} = \zeta_1 - a\frac{\partial\zeta}{\partial x} - b\frac{\partial\zeta}{\partial y} - c\frac{\partial\zeta}{\partial z}$$

With the aid of (12) we may write:

$$\zeta - m\frac{\partial\zeta}{\partial m} - n\frac{\partial\zeta}{\partial n} - p\frac{\partial\zeta}{\partial p} = \zeta_1 - (a-\alpha_4)\frac{\partial\zeta}{\partial x} - (b-\beta_4)\frac{\partial\zeta}{\partial y} - (c-\gamma_4)\frac{\partial\zeta}{\partial x} \quad . \quad (13)$$

The composition of the phase in components is represented by a, b and c; α β and γ represent the composition of this same phase in composants. In accordance with (9) the following relations are valid:

$$\begin{aligned}
\alpha(\alpha_1-\alpha_4) + \beta(\alpha_2-\alpha_4) + \gamma(\alpha_3-\alpha_4) &= a-\alpha_4 \\
\alpha(\beta_1-\beta_4) + \beta(\beta_2-\beta_4) + \gamma(\beta_3-\beta_4) &= b-\beta_4 \\
\alpha(\gamma_1-\gamma_4) + \beta(\gamma_2-\gamma_4) + \gamma(\gamma_3-\gamma_4) &= c-\gamma_4
\end{aligned}$$

When we add those three equations to one another, after having multiplied the first one with $\frac{\partial\zeta}{\partial x}$, the second one with $\frac{\partial\zeta}{\partial y}$ and the third one with $\frac{\partial\zeta}{\partial z}$, then we find, with the aid of (11)

$$\alpha\frac{\partial\zeta}{\partial m} + \beta\frac{\partial\zeta}{\partial n} + \gamma\frac{\partial\zeta}{\partial p} = (a-\alpha_4)\frac{\partial\zeta}{\partial x} + (b-\beta_4)\frac{\partial\zeta}{\partial y} + (c-\gamma_4)\frac{\partial\zeta}{\partial z}$$

With the aid of this (13) now passes into:

$$\zeta - m\frac{\partial\zeta}{\partial m} - n\frac{\partial\zeta}{\partial n} - p\frac{\partial\zeta}{\partial p} = \zeta_1 - \alpha\frac{\partial\zeta}{\partial m} - \beta\frac{\partial\zeta}{\partial n} - \gamma\frac{\partial\zeta}{\partial p}$$

which is in accordance with (4).

We may also write the four equations (5) in the form (6). For the first one of the equations (5) we may viz. write:

$$\left.\begin{aligned}
&\zeta - (x-\alpha_4)\frac{\partial\zeta}{\partial x} - (y-\beta_4)\frac{\partial\zeta}{\partial y} - (x-\gamma_4)\frac{\partial\zeta}{\partial z} \\
&= \zeta_1 - (x_1-\alpha_4)\frac{\partial\zeta_1}{\partial x_1} - (y_1-\beta_4)\frac{\partial\zeta_1}{\partial y_1} - (z_1-\gamma_4)\frac{\partial\zeta_1}{\partial z_1}
\end{aligned}\right\} \quad . \quad (14)$$

With the aid of (12) (14) passes into the first one of the equations (6).

The three equations (11) excepted, which are valid for the phase without index, we have still also three similar equations, which we obtain from (11) by giving to all variables and to ζ also, the index 1.

We call those three equations 11^a. As, however, in accordance with (5) $\frac{\partial \xi}{\partial x} = \frac{\partial \zeta_1}{\partial x_1}$ etc., it follows from (11) and (11^a) also $\frac{\partial \zeta}{\partial m} = \frac{\partial \zeta_1}{\partial m_1}$ etc.

For a ternary system with the composants F_1 F_2 and F_0 we have, when we choose F_0 as fundamental component:

$$F = m F_1 + n F_2 + (1-m-n) F_0 \quad . \quad . \quad . \quad . \quad (15)$$

When we represent the compositions of the components by α and β with the corresponding index, then the equations (9) pass into:

$$\left.\begin{aligned} m(\alpha_1-\alpha_0) + n(\alpha_2-\alpha_0) &= x-\alpha_0 \\ m(\beta_1-\beta_0) + n(\beta_2-\beta_0) &= y-\beta_0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (16)$$

We now shall deduce those equations also in another way, by which at the same time the meaning of m and n in the graphical representation becomes clear.

We take a system of coordinates with the axes OX and OY (Fig. 1) in which we represent the composition of the phases,

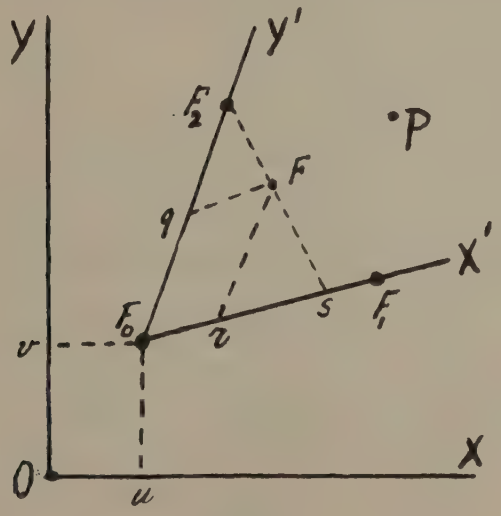

Fig. 1.

expressed in the components. We imagine the three composants F_0 F_1 and F_2 and the arbitrary phase F to be represented by the points F_0 F_1 F_2 and F. Consequently in the figure is $F_0 v = \alpha_0$, $F_0 u = \beta_0$ etc. Now we take F_0F_1 as new X-axis and F_0F_2 as new Y-axis; then the new coordinates of the point F are Fq and Fr; we put $Fq = x'$ and $Fr = y'$. When we call the angles, which the new axes F_0F_1 and F_0F_2 are making with the original X-axis φ_1 and φ_2 then it follows from the figure:

$$\left.\begin{aligned} x &= \alpha_0 + x' \cos\varphi_1 + y' \cos\varphi_2 \\ y &= \beta_0 + x' \sin\varphi_1 + y' \sin\varphi_2 \end{aligned}\right\} \quad \ldots \quad (17)$$

When we represent the length of F_0F_1 and F_0F_2 by l_1 and l_2, then we may write for (17)

$$\left.\begin{aligned} x-\alpha_0 &= \frac{x'(\alpha_1-\alpha_0)}{l_1} + \frac{y'(\alpha_2-\alpha_0)}{l_2} \\ y-\beta_0 &= \frac{x'(\beta_1-\beta_0)}{l_1} + \frac{y'(\beta_2-\beta_0)}{l_2} \end{aligned}\right| \quad \ldots \quad (18)$$

Now we shall express the composition of the phase F in that of the three composants: F_0 F_1 and F_2. We find:

quantity of F_2: quantity of $(F_0 + F_1) = Fs : FF_2$

or: quantity of F_2: quantity of $(F_0 + F_1 + F_2) = Fs : F_2s$

When we put the total quantity of $F = F_0 + F_1 + F_2$ equal to zero, and when we bear in mind that:

$$"Fs : F_2s = Fr : F_2F_0 = y' : l_2$$

then follows: quantity of $F_2 = \frac{y'}{l_2}$.

In a similar way we find: quantity of $F_1 = \frac{x'}{l_1}$.

Consequently there are wanted for forming the unit of quantity of the phase F: $\frac{x'}{l_1}$ quant. of F_1 and $\frac{y'}{l_2}$ quant. of F_2 and consequently also $1 - \frac{x'}{l_1} - \frac{y'}{l_2}$ quantities of F_0. We may write, therefore;

$$F = \frac{x'}{l_1}F_1 + \frac{y'}{l_2}F_2 + \left(1 - \frac{x'}{l_1} - \frac{y'}{l_2}\right)F_0 \quad \ldots \quad (19)$$

When we put $\frac{x'}{l_1} = m$ and $\frac{y'}{l_2} = n$ then (18) and (19) pass into (15) en (16).

Hence it appears a.o. that m and n do not represent the coordinates x' and y' of the phase F, but they are functions of them; when m and n are known, then also x' and y' are known and reversally. For this reason we may call m and n yet also coordinates.

The coordinates of the composant

F_0 are $x' = 0$ $y' = 0$ consequently $m = 0$ and $n = 0$

F_1 ,, $x' = l_1$ $y' = 0$,, $m = 1$,, $n = 0$

F_2 ,, $x' = 0$ $y' = l_2$,, $m = 0$,, $n = 1$

Of course this is also in accordance with (15); when herein we put f.i. $m=1$ and $n=0$ then phase F represents the composant F_1.

When we express the composition of a phase in its components, consequently in x and y, then x and y are positive and $x+y \leqq 1$. When, however, we express its composition in composants, then m and n may also be negative and also $m+n>1$. The latter is the case f.i. for a phase, represented by the point P. In (15) m and n are then positive and $1-m-n$ is negative.

When we have a quaternary system then similar relations exist between the coordinates viz.

$$x' = m\,l_1 \qquad y' = n\,l_2 \qquad z' = p\,l_3$$

Till now we have assumed that each of the n composants of a system of n components contains also those n components. It is apparent, however, that we may choose the composants also in such a way that one or more or even all composants contain less than n components. Of course the n composants together must contain the n components. We may consider the representation with the aid of components as a special case of the representation with the aid of composants; each of the composants then contains a single component only. We shall, however, continue by calling this a representation with the aid of components. When, however, there is at least one composant, which contains more than one component, then we shall speak of a representation with the aid of composants.

As it is known, the deduced functions of the thermodynamical potential become infinitely large when the quantities of one or more of the components approach to zero. In a quaternary system f.i. $\frac{\partial \zeta}{\partial x}$ becomes infinitely large when x or $1-x-y-z$ approaches to zero; $\frac{\partial \zeta}{\partial y}$ when y or $1-x-y-z$ and $\frac{\partial \zeta}{\partial z}$ when z or $1-x-y-z$ approaches to zero.

Using composants this is otherwise, however. It follows viz. from (11) that $\frac{\partial \zeta}{\partial m}$, $\frac{\partial \zeta}{\partial n}$ and $\frac{\partial \zeta}{\partial p}$ become infinitely large, only then when one or more of the functions $\frac{\partial \zeta}{\partial x}$, $\frac{\partial \zeta}{\partial y}$ and $\frac{\partial \zeta}{\partial z}$ are infinitely large and this may take place, as we have seen above, only when one or more of the conditions:

$$x=0 \qquad y=0 \qquad z=0 \qquad 1-x-y-z=0 \quad . \quad (20)$$

is satisfied. In general $\frac{\partial \zeta}{\partial m}$, $\frac{\partial \zeta}{\partial n}$ or $\frac{\partial \zeta}{\partial p}$ become, therefore, infinitely

large when we give such values to m, n and p, that one or more of the conditions (20) are satisfied. It is apparent that this may be casually only for $m = 0$ or $n = 0$ or $p = 0$ or $1-m-n-p = 0$.

At the same time the following is apparent. When we give to m, n and p such values that f.i. x becomes $= 0$, then in (11) $\frac{\partial \zeta}{\partial x}$ becomes infinitely large, so that $\frac{\partial \zeta}{\partial m}$, $\frac{\partial \zeta}{\partial n}$ and $\frac{\partial \zeta''}{\partial p}$ become infinitely large at the same time. When, however, we have chosen the composants in such a way that $\alpha_1 = \alpha_4$ then only $\frac{\partial \zeta}{\partial n}$ and $\frac{\partial \zeta}{\partial p}$ become infinitely large, while $\frac{\partial \zeta}{\partial m}$ remains finite.

When a liquid has the composition:

$$L = xX + yY + zZ + \ldots$$

wherein X, Y etc. represent components, then the stability requires that for all values of dx, dy etc.

$$\left(\frac{\partial \zeta}{\partial x}dx + \frac{\partial \zeta}{\partial y}dy + \ldots\right)^{(2)} > 0 \quad \ldots\ldots \quad (21)$$

When we imagine L to be divided into

$$L = \varkappa L_1 + (1-\varkappa) L_2$$

wherein:

$$L_1 = (x + dx_1) X + (y + dy_1) Y + \ldots.$$
$$L_2 = (x + dx_2) X + (y + dy_2) Y + \ldots.$$

then must

$$\zeta < \varkappa \zeta_1 + (1-\varkappa) \zeta_2$$

from which (21) is following. When we now express the composition of L in composants viz.:

$$L = m M + n N + p P + \ldots.$$

then it follows in the same way that

$$\left(\frac{\partial \zeta}{\partial m}dm + \frac{\partial \zeta}{dn}dn + \ldots.\right)^{(2)} > 0$$

must be true for all values of dm, dn etc.

(*To be continued*)

Leiden. Inorg. Chem. Lab.

Anatomy. — "*Thymus, spiracular sense organ and fenestra vestibuli (ovalis) in a* 63 *m.m. long embryo of Heptanchus cinereus*". By Prof. J. W. VAN WIJHE.

(Communicated at the meeting of September 29, 1923).

Many years ago I received this embryo from the Zoological Station at Naples. It was fixed in sublimate and preserved in alcohol. Just as another specimen it was treated with methylene blue, in order to make a skelet preparation of it.

This having proved quite successful with the one embryo, I decided to preserve the other, so as to make a series of cross sections later, in order also to be able to examine the remaining organs. I intended to wait with this until I had more of this rare material in different stages. I however received only one more embryo, 255 m.m. long, which was simply treated with alcohol and was much too large for making a series of sections. Here one would have to restrict oneself to only a few parts. For this specimen I am again indebted to the direction of the Station at Naples.

When in the autumn of 1922 I had finished with the development of the skeleton of Acanthias vulgaris [1]) I decided not to wait any longer, and a series of cross sections of the 63 mm. long embryo was made. The preservation proved to be excellent, notwithstanding the previous long treatment of HCl alcohol necessary for the elimination of the methylene blue from the remaining tissues, in order to restrict the colour to the cartilage. The staining of the sections with ammonia-carmine was also successful; but the light blue tint of the cartilage could not be intensified by the after-treatment with methylene blue or victoria blue. The reason for this remained unknown to me.

In the 255 m.m. long embryo, which had been in alcohol for many years, the cartilage suffered itself to be stained deep blue.

[1]) VAN WIJHE, J. W. Frühe Entwicklungsstadien des Kopf- und Rumpfskeletts von Acanthias vulgaris. Bijdragen tot de Dierkunde, publ. by the Kon. Zool. Genootsch. Natura Artis Magistra at Amsterdam. Afl. 22, Feestnummer voor MAX WEBER, 1922.

1. *Thymus.*

The development of the thymus in the Selachians was first described by Dohrn (1884). The facts then found by him were principally confirmed by later investigators. Hammar, who had given many years to the study of the structure, development and function of this organ in nearly all the principal groups of vertebrates, described the development in the Selachians in 1912, and gave a detailed account of the results of his predecessors.

He found, that in all vertebrates from fish up to man, the thymus continues to grow till the time of puberty. Then an involution period begins, wherein it as a rule atrophies, without totally disappearing.

The thymus, in all vertebrates, begins to form as a local proliferation of the epithelium of the gill clefts.

In man it appears principally, if not exclusively, on the third gill cleft, but in the Selachians, which generally have six gill clefts, a beginning of the thymus is described on each gill cleft. These however speedily disappear on the first and last, sometimes even on the last two gill clefts.

Not all investigators are of opinion that the thickening of epithelium cells of the first gill cleft (spiracle) may be considered as a thymus, and it is possible that here an interchange may have taken place with the place of origin of the spiracular sense organ.

Soon after its appearance, one can distinguish in the thymus two different kinds of cells, viz. a network of flat epithelial cells, which encloses groups of round cells in its meshes.

These round cells multiply themselves so quickly, that the network can no longer be discerned unless in very thin sections.

The whole organ, which formerly was pear-shaped and afterwards has the shape of a grape bunch, appears to be wholly constituted of round cells, which form a solid mass without lumen. These cells hardly have any protoplasm, and therefore give the appearance as if one only has to do with an accumulation of nuclei.

There are two opinions concerning the derivation of these round cells, which strongly resemble the lymphocytes of the blood. Many hold them for epithelium cells, which have rounded themselves off; others again take them to be true lymphocytes, which have penetrated the organ from the bloodvessels and the neighbouring mesenchym. The latter opinion is emphatically upheld by Hammar for all classes of vertebrates.

The question as to which of the two opinions is correct, cannot

be settled by the study of the 63 m.m. long embryo of Heptanchus, but a further question can be explained thereby, viz. whether the thymus has to be considered as a gland which has lost its original excretory duct and thus only has internal secretion left. It would then find itself in a similar condition as the anterior lobe of the hypophysis and the thyroid gland, which, however, in the embryo of vertebrates, always have an excretory duct which is only lost during the further course of development.

The thymus does not sever itself from the epithelium of the branchial gut in Cyclostomes and most of the bony fishes. This is however the case with the remaining vertebrates. But a true excretory duct, as a rule, does not appear. This would be expected in sharks, but FRITSCHE (1910) says: "Ein Lumen und einen Ausführungsgang habe ich bei Spinax ebensowenig auffinden können wie DOHRN bei seinen Haifischen."

In a very early stage of rays (Torpedo), they however noticed something which resembled an excretory duct.

In some of the sharks examined up to now, the body of the thymus separates itself directly, without a pedicle, from the epithelium of the branchial gut; while in others it still remains connected for some time by a stalk to the epithelium.

This stalk lacks the characteristics of an excretory duct, because it not only has no lumen, but also shows the same structure as the body of the thymus and consists almost exclusively of the rounded cells, which resemble lymphocytes.

In our embryo of Heptanchus we on the contrary find an excretory duct *in optima forma* for each of the thymus divisions (thymomeres) which are found on both sides of the body, one for each side from the second to the seventh branchial cleft. There are 8 gill clefts, but in the first (spiracle) and last the thymus is absent.

It is the largest in the second and third cleft and has the form of a bunch of grapes. The bunch is smaller in the 4th cleft, in the 5th still smaller, and in the 6th the thymus no longer has the bunch form, but is composed of a single acinus, into which the excretory duct opens.

In the 7th cleft every acinus is found missing from the short excretory duct.

In the figure of the section we see the large thymus of the 2nd branchial cleft. It runs over the top of the 1st epibranchial and then continues as the fairly long excretory duct. This has an obvious lumen, which with its one end opens at the top of the branchial

cleft, with the other reaches to the body of the thymus without entering it.

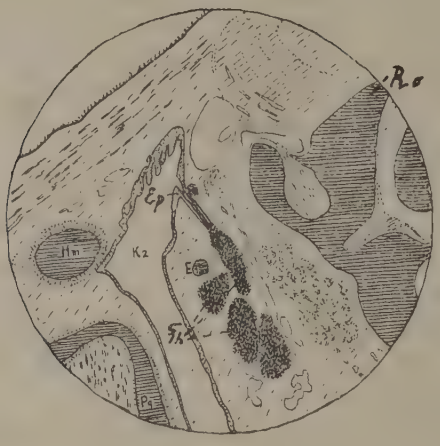

Fig. 1. Cross section through 2nd branchial cleft of a 63 m.m. long embryo of Heptanchus cinereus. In this and in the following figs. the cartilage (stained blue in section) is striated horizontally.

The wall of the duct is 2 cells thick, and is constituted of a double layer of fairly flat epithelium cells, amongst which not a single round cell is to be found.

The excretory duct of each of the remaining thymomeres shows a similar structure, viz. a double layered epithelial wall, encircling a lumen, which opens into its respective gill cleft.

These ducts from the 2nd caudally, gradually shorten; the last (6th) forming a rather unimportant, yet distinct attachment to the 7th branchial cleft.

The excretory ducts are not permanent. They later on lose their epithelial structure and lumen. This e. g. happened in the 225 m.m. long embryo. Here, in the place of the excretory duct of the first (anterior) thymomere, one finds a long pedicle, which appears as an outgrowth of the thymus. The pedicle runs over the top of the 1st epibranchial and reaches the wall of the branchial cleft.

It shows itself as a chord, which appears entirely to consist of lymphocyt-like round cells. No traces are left of the original epithelial structure and lumen. I however do not wish to deny the presence of a reticulum. It would also be possible to make it clear in the pedicle by appropriate methods.

For completeness the so called epithelial bodies and the suprapericardial organ should also be mentioned. In the 63 m. m. embryo an epithelial body is found, immediately above the opening of the 1st and 2nd thymomere. Each little body is a round isolated cellmass, which resembles an acinus of the thymus in form and

size, but is more compact; owing to the fact that it has finer lymph-spaces than the thymus. No trace of such a body was to be found at the 3rd, 4th, 5th and 6th thymomere.

The suprapericardial body was discovered by VAN BEMMELEN [1]) (1885) at the end of the branchial gut. Later it was found in all classes of vertebrates. It is generally taken as the last indication of an abortive branchial pouch, and mostly appears on only one side of the body.

In 1906 BRAUS found it in the 67 m.m. long embryo of Heptanchus, which very likely originates from the same mother-animal as mine, and I can corroborate his statement. It is only well developed in the left half of the body, and shows itself as a little bladder, the lumen of which is encircled by a single layer of fairly columnar epithelium cells. It is to be seen on 35 sections, and is situated as BRAUS stated, behind the last visceral arch, in the angle which this makes with the ceratobranchial. Just as BRAUS, I found it near its posterior margin connected with the epithelium at the base of the branchial gut by a short pedicle.

On the right side the organ is rudimentary.

I found it represented by a flattened little group of epithelial cells without a lumen, and totally severed from the gut epithelium. This is visible in the sections passing through the posterior half of the vesicle on the left. BRAUS does not mention this little group.

His specimen was probably somewhat further developed than mine,

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[1]) Owing to the presence of a suprapericardial body in the embryos of Heptanchus (VAN BEMMELEN in vain sought for it in the adult animal) one cannot assume that, in higher animals, this little body is the remains of a branchial cleft, which is present in the Notidanides as such. The morphological significance of this organ is a problem. One may of course believe that it is the remains of a branchial cleft, which still lies further caudally than the last (8th) of Heptanchus. BRAUS e. g. takes it to be the rest of a 10th branchial pouch.

He professes to find the remains of a (9th) branchial pouch in a slight protrusion of the intestinal wall behind the last branchial arch, in the angle between the last (7th) ceratobranchial, and a caudalwards directed protuberance on its ventral side.

Although this protuberance chondrifies continuous with the 7th ceratobranchial, he considers it to be the remains of an 8th branchial arch.

I cannot agree with these conceptions. In my specimen the rather long protuberance is still quite prochondral, and just like the prochondral cardiobranchial end, lies in the beginning of the oesophagus. In the protuberance I can only discern a processus muscularis of the 7th ceratobranchial, morphologically insignificant. An intestinal protrusion which could also be considered as a 9th branchial pouch, is not present, and I must consider it as an artificial product in the specimen of BRAUS.

and this little group more atrophied. He thought he saw an indication of an antimere of the left vesicle on the right side of the body, in the shape of a more caudally situated diverticulum of the branchial gut.

Let us however return to the thymus. The *genus* Heptanchus is indeed rightly regarded as the most primitive of the living Selachians. The number of visceral pouches (i. e. 8) surpasses that of all other fishes and higher animals. Only the anterior 5 are still formed in mammals.

Concerning the 63 m.m. long embryo of Heptanchus, we may now assume, that also its thymus appears in a more primitive form than in the development of higher animals.

The original function of the thymus could then not have been internal secretion only, but it must also have removed products through its excretory ducts.

Originally each thymomere was a true gland, according to the old notion, with an excretory duct even as was the case with the thyroid and the anterior lobe of the hypophysis.

The presence of excretory ducts is also of importance for the conception of the morphological significance of the gland. Since the researches of Dohrn, it is generally accepted that the thymus is a branchiomere organ, a division of which occurred on each branchial cleft.

Now Amphioxus has on each of its many branchial clefts a glandular body, which opens with its excretory duct into the top of the cleft. This branchionephros functions as an excretory organ, and for many years I have presumed, that it would prove homologous to the thymus of higher animals.

This presumption was strengthened, when in 1909 Goodrich found that the branchionephros does not develop from the coelomic epithelium, as one would rather be inclined to assume for an excretory organ in chordates.

But he does not state that it develops from the branchial epithelium. His drawings however give this impression. Might this impression prove to be correct by later investigations, then the branchionephros develops from the same tissue as the thymus of higher animals. Cells resembling lymphocytes are never found in it. Lymphocytes do not occur in the blood of Amphioxus, the blood of which only consists of plasma, without any red or white blood corpuscles, just as the blood in its earliest stage in craniates.[1])

[1]) A few investigators profess to have found cells in the blood of Amphioxus. I have never observed any in my numerous sections of larvae and adult animals.

The presumed homology of the thymus and branchionephros has also been supported from the side of the craniates, now that, in the development of such a primitive form as Heptanchus, the presence in the thymus of excretory ducts, which in Amphioxus analogously open into the branchial clefts, has been shown.

If the branchionephros develops from the branchial epithelium, the chief difficulty to homologize it with the thymus, I think then lies in the period of development of this gland. One should expect the thymus to become perceptable in a very early period of its development, but this only happens very late.

The reason for this is because the original function no longer comes to development. It is taken over by the pronephros and the mesonephros. The other function of the thymus i.e. its internal secretion, caused by the lymphocytlike cells, must phylogenetically have originated much later.

2. *Spiracular sense organ.*

In no vertebrates does a division of the thymus come to development in the first branchial cleft (spiracle). It appears not even to be formed there at all. On the other hand, we find on the wall of the spiracle in the embryos or larvae of the more primitive fishes: Selachians, Ganoids and Dipnoi a sense organ, which is not met with on any of the remaining branchial clefts. These adult fishes also possess one.

We find it even in those forms (Dipnoi and Holostei) in which the spiracle, which is developed in the manner of an intestinal pouch, no longer breaks through outwardly.

It was discovered by Ramsay Wright in 1885, who found it as a protrusion of the medial wall of the spiracular visceral pouch of the Holostei (Lepidosteus and Amia). This protrusion (diverticulum) is directed upwards and surrounded by the cartilaginous auditory capsule; in other words, it lies in a canal of the lateral cartilaginous wall of the otic region of the skull, but otherwise has no relation to the auditory organ.

A similar canal in the cranial cartilage, into which a diverticulum of the spiracular wall penetrates, was discovered by Bridge in Polyodon. The same was also observed by Wright in the sturgeon. The presence of a sense organ in these Chondrostei is, however, not mentioned.

Wright found, that in the Holostei this sense organ is innervated by a branch of the ram. oticus of the facial nerve, which in the

Ganoids (Chondrostei and Holostei) is likewise overgrown by the cartilaginous auditory capsule, and of which (ram. oticus) it was known that it sends out branches in this region to the sense organs, belonging to the lateral line system.

These sense organs, called neuromasts (Nervenhügel) by WRIGHT, lie either free on the surface, or protected in little sacs, grooves or canals; all are of ectodermal derivation. Now it was noteworthy that the sense organ of the spiracular pouch also resembled the structure of a neuromast, although WRIGHT evidently thought it to be of entodermal origin. It seemed as if one here had the unexpected example of a sense organ of the Chordates, which did not originate from the ectoderm, although it was still supplied by a nerve, belonging to the lateral line system of the epidermal sense organs.

The study of the Dipnoi dispelled the singularity of this phenomenon. In this group PINKUS (1895) discovered in Protopterus annectens a little bladder with a sense organ on its wall, and imbedded in the cartilage of the otic region. The sense organ — evidently a neuromast according to the fig. — is supplied by a caudalwards running branch of the facial nerve, the branch belonging to the lateral line system.

PINKUS still describes two more caudalwards running branches from the lateral line system of the n. facialis. The one forms the well known anastomosis with the ramus lateralis vagi (and glossopharyngei) the other he calls ram. oticus. He, however, draws the origin (l.c. fig. 3) of these branches so close to each other that, according to my opinion, one has to consider them as the strongly developed homologue of the ram. oticus of the Ganoids.

Of this organ PINKUS says (l. c. p. 307) "Das Organ ist zweifellos ein Derivat des Seitenkanales. Ueber seine Bedeutung vermag ich übrigens nichts auszusagen, da vergleichend anatomische und entwicklungsgeschichtliche Thatsachen mir bisher fehlen".

For the knowledge of the development we are indebted to AGAR, (1906) who examined the first stages of the spiraculum in Lepidosiren and Protopterus.

He showed that this sense organ is of ectodermal origin. This seat of origin reaches the top of the solid gut protuberance, which represents the spiracle, and then severs itself from the ectoderm. The organ then naturally gives the impression of having been derived from the entoderm.

AGAR like PINKUS, was not aware of the work of RAMSAY WRIGHT, otherwise he would undoubtedly have mentioned, that the presence of a spiracular sense organ in Holostei was already known. He also

would not have neglected to point out, that, in the Holostei, we have no reason to believe in the entodermal origin of the sense organ, now that in the Dipnoi [1]) its formation from the ectoderm is manifest.

As opposed to PINKUS, AGAR says "This organ has no relation to the lateral line system of sense organs". To my opinion, however, it undoubtedly belongs to this system, because it possesses a neuromast, is supplied by a branch from the lateral line system of the facial nerve, and moreover is clearly of ectodermal origin in the Dipnoi.

The majority of epidermal sense organs, sinks under the epidermis during the ontogenetic period, and finds protection by the subcutaneous connective tissue. Only one organ having its seat of origin in the immediate vicinity of the spiracle, sinks therein, acquiring a considerable development.

In my opinion this not only happens when the spiracle no longer breaks through outwardly, retaining its opening into the gut, as in the Holostei, but also, when it moreover loses its connection with the gut, as in the Dipnoi.

Let us now proceed to the Selachians. In these WRIGHT examined the spiracle of a 60 m.m. long embryo of Mustelus. Here he found two diverticula, situated above each other, on the medial wall. The dorsal diverticulum reached till under the canalis semicircularis lateralis of the auditory organ, and was already discovered in a number of adult Selachians, by JOH. MÜLLER (1841).

The ventral diverticulum did not reach the cranial cartilage, and at one place contained columnar epithelium, which he took for sense organ epithelium, and which according to him, was supplied by the ram. praetrematicus of the facial nerve. This innervation would lead us to expect, that we have here to deal with a different sense organ to that in the Holostei. PHELPS ALLIS, however, in 1901, examined a 122 m.m. long embryo of Mustelus, and was able to trace the nerve from the organ till near the ram. oticus, the same branch which also supplies the sense organ in the Holostei.

Independent of WRIGHT'S work, that of VAN BEMMELEN appeared in the same year (1885). The latter, besides in Mustelus, found both the diverticula in a great number of Selachians, in embryos as well

[1]) GREIL (1913) mentions the ectodermal origin of the sense organ ("Hyomandibular organ") in Ceratodus, and its innervation by a branch from the lateral line system ("ram. hypoticus") of the facial nerve. Whether the sense organ in Ceratodus is afterwards also surrounded by the cranial cartilage, I do not find mentioned.

as in the adult fishes. He found both (the dorsal and the ventral) simultaneously in the same animal, in the forms which now-a-days, after TATE REGAN, are called Galeoidei. In rays on the contrary, only the ventral diverticulum of the examined fishes: Raja, Torpedo, Trygon and Myliobatis was found to be present. The dorsal one was absent in concurrence with the results of JOH. MÜLLER, who found it in rays only in the family of the Rhinobatidae.

Vice versa the ventral diverticulum was found missing, while only the dorsal one was present in Acanthias and Heptanchus; each of which is a representative resp. of the groups Squaloidei and Notidanoidei.

On the ventral diverticulum a follicle, resembling an oval bladder, develops in all forms which possess it. It nearly touches the auditary labyrinth, is lined on the inside with columnar epithelium, and is connected to the wall of the spiracle by a pedicle, which may, or may not have a lumen. In an adult Torpedo the bladder was found to be very large.

As regards the morphological significance of the follicle, VAN BEMMELEN thought of the probability of a homologue with the suprapericardial body, which primarily is also a single little bladder. He says (l. c. p. 178) "[später] tritt aber der grosse Unterschied ein: die Suprapericardialkörper entwickeln sich zu drüsenartigen Gebilden [1]) die Spritzlochbläschen treiben nur eine oder zwei acinöse Ausstülpungen oder bleiben wohl ganz einfach."

VAN BEMMELEN further thought of the probability of considering the follicle, even as the suprapericardial body, as the remains of an original gill cleft.

My opinion is that this conception cannot be adhered to any longer, and that the follicle is a spiracular sense organ bladder.

VAN BEMMELEN did not consider this possibility, because he had evidently not observed a supplying nerve.

No mention is made of the appearance of a follicle from the dorsal protrusion of the spiracle in the Galeodei. We may thus accept that it is absent there.

Acanthias and Heptanchus only show the dorsal protrusion. Is the spiracular sense organ now also found missing in them or not?

VAN BEMMELEN speaks of a "dorsale Ausstülpung", but also calls it an "Anhang" of the spiracle. He says: (l. c. p. 176). "Bei erwachsenen Exemplaren von Acanthias endlich konnte ich den Anhang

[1]) Their structure in the Selachians, then has much in common with that of the thyroid gland, from which they, however, totally differ morphologically.

als ein sackförmiges, ungefähr 3 m.m. langes Gebilde aus dem Bindegewebe frei präpariren, seine Wände zeigten sich ausserordentlich dicht und inwendig glatt, das Epithelium hoch und drüsig. Ebenso zeigte sich der dorsale Anhang von Heptanchus, aber relativ noch kürzer". As it will presently be seen, he undoubtedly dissected out the sense organ bladder.

HOFFMANN (1899) inter alia also investigated the development of the diverticulum of the spiracle in Acanthias. He found it to make its appearance first in 28 m.m. long embryos and innervated by a branch from the lateral line system of the facial nerve.

He considers this branch, which also supplies epidermal sense organs, most likely homologous to the ram. oticus of the Ganoids. The diverticulum is soon directed forwards with its blind end, and unites itself there with the nerve. I can confirm this from my material of Acanthias.

HOFFMANN discovered the innervation, well knowing of the work of WRIGHT, from which he quotes in detail. He, however, missed the conclusion that a sense organ had to be present. He was too much under the impression of having here to do with the vestigial part of a branchial pouch, which had disappeared.

Besides the two embryos of Heptanchus, my own investigation also includes a series of sections (15μ thick) through embryos of Acanthias varying in length from 23 to 98 m.m.

In the 23 m.m. long embryo, the anterior wall of the spiracle forms a rostrally directed diverticulum, next to the auditory organ, from which it is separated by the jugular vein (the nervus facialis running under the vein). The diverticulum is to be seen on 7 sections anterior to the external opening of the spiracle, and has the shape of a cone flattened on one side, the axis of which runs parallel to the longitudinal axis, passing through the notochord. The three anterior ones of the seven sections pass through the top of the cone, which is distinguished by its columnar epithelium, so that the lumen appears for the first time on the third section. One also sees the termination of the branch of the ram. oticus connected here to the group of the columnar cells. HOFFMANN already pointed out, that one could stipulate, through this connection the situation of the organ before it is more clearly defined.

A cross section through the anterior margin of the external opening of the spiracle on the skin at the same time passed through the internal opening towards the intestine in an embryo of 39½ m.m. of which I in 1922 described the skull. The diverticulum is to be seen on 21 sections rostralwards. Just as in the embryo of 23 m.m.

it runs forwards along the auditory capsule and is separated from it by the jugular vein and the facial nerve [1]).

If we trace the diverticulum from the base rostrally, we see it after 8 sections already changed into a flat and narrow duct with a lateral and medial wall. The duct is prolonged over 4 sections, and then with nearly no change of lumen, passes over into the top part of the diverticulum, which is perceptable on 9 sections. The medial wall of this part has over its whole length a neuromast, whose posterior end is clearly defined. Near the rostral end (the blind top) of the diverticulum the branch of the ram. oticus unites with the neuromast.

We may now, proceeding from the anterior margin of the spiracle, distinguish three parts, seen resp. on 8, 4 and 9 sections which we shall call vestibulum of the spiracle, excretory duct and corpus of the sense organ bladder.

Excretory duct and corpus are partners, but the vestibulum is nothing more than an ordinary diverticulum of the anterior wall of a visceral pouch, and disappears later, in consequence of the enlargement of the external opening of the spiracle.

The vestibulum is still present in an embryo 69 m.m. long, but in embryos of 78 m.m. or more, it has disappeared. We then only see on a section, passing posterior to the anterior margin of the spiracle, the opening, which meanwhile has become very minute, of the excretory duct. Then the condition of the sense organ bladder principally corresponds to that of the organ which occurs in the adult animal. It then forms an appendix of the spiracle. The description by VAN BEMMELEN of the Galeoidei and rays also applies to the sense organ of Acanthias.

Probably these bladders are homologous in all the Selachians and of ectodermal origin. They have in some forms sunk somewhat deeper into the spiracle, than in others. We shall still examine the little bladder sowewhat closer in a series of cross sections of the Acanthias embryo 98 m.m. long.

The very minute opening in the anterior wall of the spiracle is only to be seen in one section. From here the organ passes rostralwards over 50 sections. It runs along the auditory organ from

[1]) During the translation of this paper I prepared a series of sagittal sections, stained with haematoxylin and eosin, of a 22 m.m. long embryo of Torpedo marmorata. I found the deep neuromast at the inner wall of the spiracle innervated by a branch of the ram. oticus, crossing the outer side of the vena jugularis, just as in Acanthias.

which, — as previously — it is seperated by the jugular vein and the facial nerve.

The corpus of the bladder, with its long neuromast, is visible on the anterior 21 sections. The excretory duct falls in the following 29 sections. The neuromast thus nearly constitutes half the length of the organ, and is much larger than in the lateral line system organs of the skin. Round the corpus one sees the mesenchym in more compact formation, the first stage of a connective tissue capsule. The excretory duct, immediately posterior to the corpus, shows a different construction than further caudalwards.

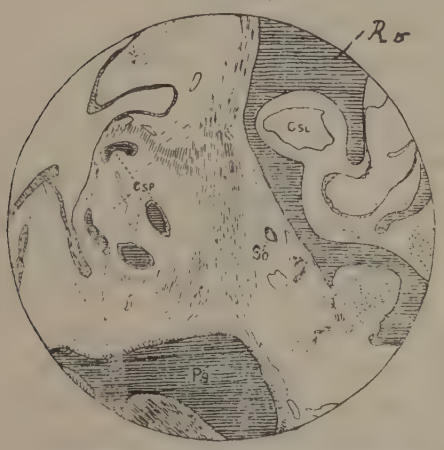

Fig. 2*a*. Cross section through the otic region of the skull and the anterior wall of the spiracle, from a 98 m.m. long embryo of Acanthias vulgaris.

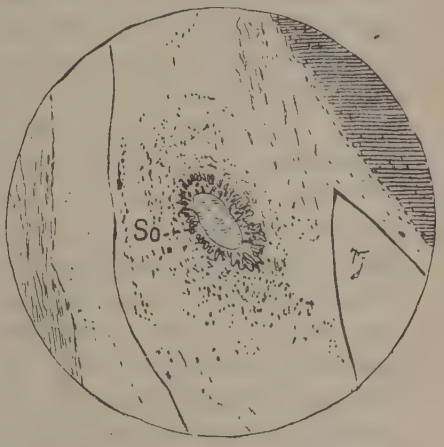

Fig. 2*b* shows the spiracular organ under high power. Its contents, mucus (stained blue in section) are seen as thin striations.

On the first 5 sections behind the corpus, the medial wall of the duct is thickened, as the result of the proliferation of the outer layer of epithelium cells. Here the oval lumen is wider than in other places. The longitudinal axis of the oval is more or less twice as long as in the corpus. On the following 24 sections this lumen continually decreases, the wall consisting of two layers of cells. Those of the inner layer are very flat, those of the outer layer may be called cubic.

It is of importance that the corpus of the sense organ bladder and the proximal part (5 sections) of the duct, should be filled with mucus, which in this stage (and later) allows itself to be stained blue, just as in the ampullary and canal organs of the lateral line system. In the distal part of the duct (24 sections) the mucus is present in lesser quantity.

From this we may see, that the spiracular sense organ shows itself to belong to the lateral line system of epidermal sense organs, which is generally also understood by the term mucus-organs. The direct proof has not yet been given, but may perhaps be found in stages earlier than those which I have studied.

The ram. oticus, in all the studied embryos, arises with a ganglion like thickening from the buccal ganglion of the facial nerve.

In the 39½ m.m. long embryo, it runs along the cartilage of the ear capsule — but not yet surrounded by the cartilage — dorsally and caudalwards. It sends off a few thin branches to the organs in the lateral line canal of the regio otica, and a thick branch, which goes to the spiracular sense organ across the jugular vein.

In the 98 m.m. long embryo, a part of the ram. oticus is overgrown by the cartilage of the ear capsule. This is also the case with the Ganoids. Contrary to the Selachians the sense organ itself is surrounded by cartilage in both Ganoids and Dipnoi.

We shall now pass on to the 63 m.m. long embryo of Heptanchus. The small external opening of the spiracle is here situated far backwards. The fissure like opening in the gut reaches still further rostralwards. If we accept that the beginning — the base — of the vestibulum falls on the section which passes through the anterior margin of this fissure, then the top of the vestibulum lies still 28 sections further forwards. In this top the sense organ bladder opens without an excretory duct. It can be traced in 12 sections rostralwards, along the auditory organ, from which it is separ-

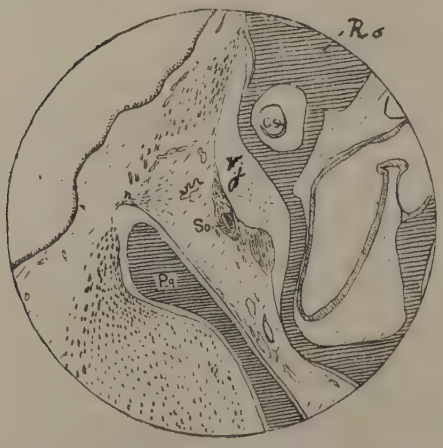

Fig. 3*a*. Cross section through the otic region of the skull of a 63 m.m. long embryo of Heptanchus cinereus.

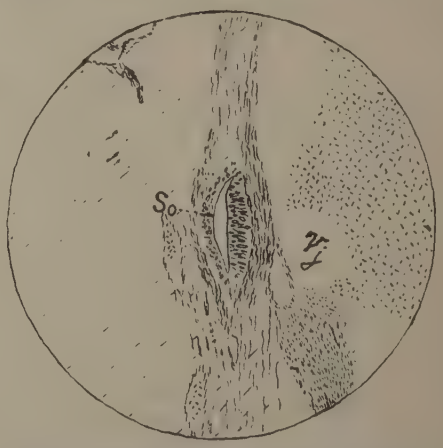

Fig. 3*b* shows the spiracular organ under higher magnification.

ated by the jugular vein. The neuromast on the medial wall just projects with its posterior margin from the vestibulum.

I was not successful in finding the supplying nerve. Perhaps it is owing to the intensely stained connective tissue capsule, which is more developed than in the largest of the examined embryos of Acanthias. In the 225 m.m. long embryo the organ was so badly preserved, that nothing of importance can be mentioned [1]).

3. *Fenestra vestibuli.*

In the 63 m.m. long embryo of Heptanchus, the attachment of the hyomandibular to the auditory capsule is brought about by a thin layer of connective tissue, wherein I can find no cavity of

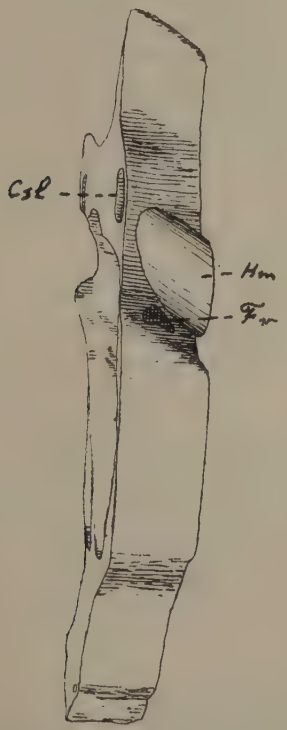

Fig. 4. Lateral surface of the model of a disk from the cartilage of the regio otica of an embryo of Heptanchus cinereus. The disk is placed in such a position that a part of the anterior surface with the canalis semicircularis lateralis, is just visible, and the fenestra vestibuli is not covered by the upper part of the hyomandibular.

[1]) Before the translation of this paper, the work of VITALI (Anat. Anzeig. 1911 and 1912) had escaped my notice, and I am indebted to Dr. BENJAMINS of Utrecht for having called my attention to it. As he remarks, this paratympanic organ in birds must be the homologue of the spiracular sense organ. An interesting referate of the works of VITALI on this organ by RUFFINI "Sull organo nervoso paratimpanico di G. VITALI od organo del volo degli uccelli" is to be found in "Archivio Italiano di Otologia Rinologia e Laringologia" publ. by GRADENIGO. Vol. 31, 1920.

articulation. It is prolonged over 49 sections, 15 μ thick. Immediately ventral to the anterior portion of this place of attachment one sees in the sections 5, 6 and 7 (in antero-posterior sequence) a connection through a small opening in the wall of the auditory capsule, between the mesenchym which in this stage fills the perilymphatic space, and the mesenchym outside the capsule. The posterior margin of the opening is not clearly defined, so that it remains dubious whether the hole is present in the next three sections or not. On the contrary the margins of the opening in the 255 m.m. long embryo, are clearly defined. The attachment of the hyomandibular to the capsule takes place here on about 59 sections 30 μ thick (in all the other embryos the sections are 15 μ thick).

The opening reaches from the 8th to the 25th section (counted antero-posteriorally). It is closed by a deeply red stained connective tissue, which also helps to connect the hyomandibular to the skull, and which is rather conspicuously surrounded by the blue colour of the cranial cartilage. The opening lies in the under part of the fossa for the hyomandibular, which partly covers it.

From the wax model of Mr. P. J. DE VRIES, made according to the method of BORN, one can see that the opening is not truly oval, but rather kidney-shaped, because the under margin forms a re-entering concavity. The mesenchym which formerly filled the perilymphatic spaces, has to a large extent disappeared and been replaced by a liquid, which is prevented from flowing out, by the connective tissue closing the opening.

The opening, owing to its position, has to be considered as the homologue of the fenestra vestibuli, which in Amphibians and Amniotes is closed by the stapes, and which according to general opinion would be absent in fishes.

Owing to the great length of the embryo, it must have been more or less fully developed, and it is improbable that the fenestra would not persist after birth.

I, however, had no opportunity of examining adult material. Irrespective of the autostylic Dipnoi and Holocephali, fishes are as a rule hyostylic. Their powerful hyomandibular functions in the first instance as a suspensorium. This fact evidently has to do with the absence of a fenestra vestibuli. Only two primitive forms viz. Heptanchus and Hexanchus are amphistylic. Their hyomandibular, owing to the firm attachment of the palatoquadrate to the skull, can only feebly function as a suspensorium. It is therefore conceivable, that the hyomandibular, at least in Heptanchus, may still have the function of transferring vibrations to the auditory organ.

The presence of the fenestra in the embryo is in any case a support to the old theory, which in later years has frequently been attacked, the theory namely: that the stapes in higher animals is homologous to the hyomandibular in fishes.

INDEX LETTERS.

Csl. Canalis semicircularis lateralis.
Csp. Cart. spiracularis. Each of the two spiracular cartilages (fig. 2*a*) is sectioned twice.
E. Top of the epibranchial of the first branchial arch.
Ep. Epithelial body.
Fv. Fenestra vestibuli (ovalis).
Hm. Hyomandibular.
K_2. Second branchial cleft.
Pq. Palatoquadrate.
Ro. Regio otica of the skull.
So. Spiracular sense organ.
Th. Thymus.
Vj. Vena jugularis.

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Physiology. — "*Contributions to an experimental phonetic investigation of the Dutch language. I. The short o*". [1]). By Miss L. KAISER. (Communicated by Prof. G. VAN RIJNBERK).

(Communicated at the meeting of September 29, 1923).

When listening carefully to the pronunciation of the "o" in closed syllables in Dutch, we perceive that — apart from the influence which all sounds undergo from preceding and following vowels or consonants — two completely different ways of pronunciation can be distinguished [2]).

One of these two pronunciations is heard in words like: kok, tot, hol; the other in words like pop, bot, vol, hond. At the suggestion of my former master Dr. PLOMP, I have tried to go further into this question.

I first tried to determine experimentally this difference, suggested by linguistic feeling and observed by simple hearing.

Experimental phonetic analysis of the speech movements.

Several methods used in experimental phonetics were consecutively applied in order to determine the essential movements and positions of the vocal organs during the pronouncing of o̜ and o̩ [3]). In doing so I chiefly made use of one trial person, while the results were afterwards tested to those obtained with other speakers.

1. Observation and measuring of the mouth opening while pronouncing different sounds proved that in this respect a, o̜, oo, o̩, oe form a series in which the mouthopening gradually decreases, the height

[1]) From investigations made at the Physiol. Lab. of the Amsterdam university and at the Phonet. Lab. of the Czech university at Prague.

[2]) I am aware of the fact that so called educated speech varies considerably in different parts of this country. As far as I know facts mentioned here hold good for the pronunciation of Amsterdam and surroundings and probably not or only partially e. g. for that of the Hague and surroundings in which the o-sound seems to predominate.

[3]) The o of kok is represented by o̜, that of pop by o̩.

diminishing regularly, while the width also decreases but not so regularly. The latter, namely, shows a sudden decrease between o̤ and oo. In this series the height of the mouth opening was 16 mM., 12 mM., 8 mM., 6 mM., 4 mM. respectively, the width 36 mM., 31 mM., 16 mM., 14 mM., 7 mM. respectively (see fig. 1).

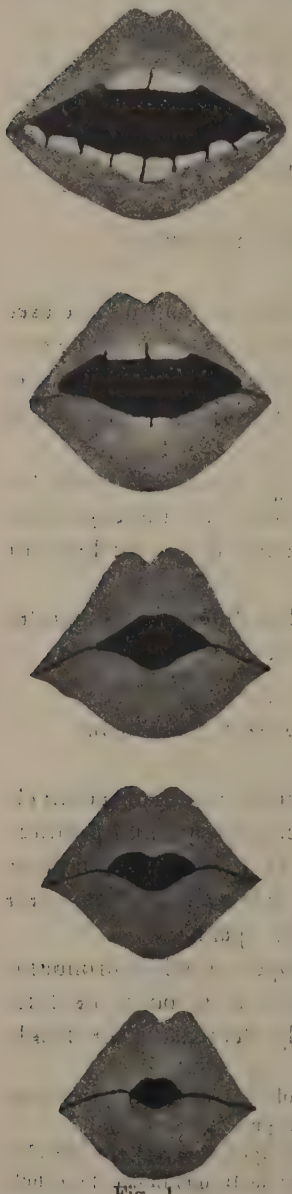

Fig. 1.

Closely connected with this are curves of the lipmovements made with the apparatus of VON WILCZEWSKI[1]). This apparatus has been so construed as to have the curves indicate the natural size of the vertical lip-opening. Fig. 2 illustrates this. Fig. 3 shows a curve obtained with the same apparatus by pronouncing alternatively dọl and dọ̤l. The difference is clear; the dimensions are about the same as those mentioned above.

Consequently, if we exclusively consider the shape of the mouth opening, we can imagine that o̤ is an oo that became more or less like an a while ọ is an oo that has acquired some of the qualities of the oe.

2. By means of ZWAARDEMAKER's apparatus[2]) for registering speech movements, the pouting of the upper lip, the movements of the lower jaw relative to the upper jaw, and the contraction of the muscles that form the bottom of the mouth, were recorded. Fig. 4 shows that also as regards jaw opening a, o̤, oo, ọ, and oe form a descending series, while the pouting of the lips increases, (with this trial person there is less pouting of the lips for ọ and oe than for oo in connection with the downward movement of the upper lip, during which the latter is sowewhat flattened). The curve of the mouth bottom is not dealt with here because of its complexity. What interests us most in this curve is that it shows considerable and characteristic dif-

[1]) Vox. Heft 3/6, 1922.

[2]) Onderz. Physiol. Lab. te Utrecht. Ve reeks I 1899—1901 p. 76. Leerb. II p. 98.

ferences between the two o-sounds. These results harmonize quite well with those obtained by EYKMAN [1]) who, working with the same instrument, found an average jaw opening of 7,25 mM for a

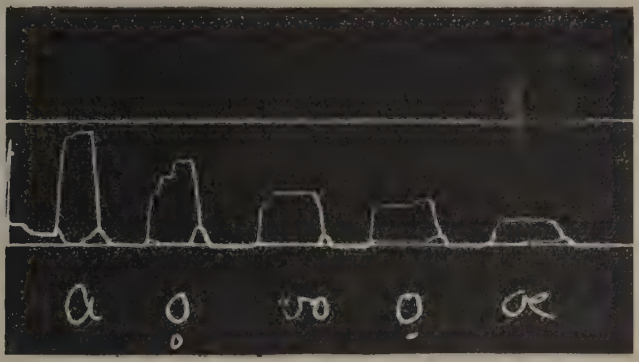

Fig. 2.

in "bat", 5,50 mM for o of "pot", 4,75 mM for oo of "boot", 4,50 mM for o of "bot" and 2,25 for oe of "boet".

Fig. 3.

Fig. 5 also shows curves of lip, jaw, and mouth bottom, but these curves are obtained in another way, viz. by means of a "mouth-funnel" that permits of registering the above mentioned movements at the same time. This instrument, constructed by me for another purpose, will be dealt with elsewhere. As it has no fixed support, it misses the exactness which characterizes ZWAARDEMAKER's apparatus. Still it is very useful to give a provisional impression of something relative. It can be noticed in fig. 5 that in pronouncing "dorscht" there is less pouting of the lips and a larger jaw opening than for „dorst", while the curve of the mouth bottom is almost the same for both words. From the mouth-funnel curve it appears that the air current for o is stronger than for o, as is easily comprehensible. From the above, therefore, it becomes evident again that the two sounds differ considerably.

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[1]) Onderz. Physiol. Lab. te Utrecht Ve reeks II 1899—1901 p. 202.

3. With the majority of speakers the hard palate is either hardly touched or not touched at all by the tongue in pronouncing oo, o̥, or o̞. Consequently the artificial palate cannot be of much use here. Yet I had the words "po̥p" and "bo̥b" pronounced by two trial persons with whom a rather large part of the palate was touched. The results can be found in fig. 6. The difference between the two sounds is clear with both persons: the surface touched for o̥ being smaller than for o̞, while it is a wellknown fact that for a the palate is not touched at all.

4. Finally the movement of the larynx was registered. It can be easily felt that the larynx assumes a somewhat different position in the two cases, viz. it is advanced more for o̞. However, I did not always succeed in recording this difference. I tried to do so with ZWAARDEMAKER's method [1]). The curves obtained, however, were too unlike in appearance but that definite conclusions could be drawn. Still it appeared from these curves that the larynx was retracted for o̥ (as for a and oo), while it was advanced for o̞ as for oe, though by no means to such a degree. Fig. 7 shows part of a curve in which the difference between o̥ and o̞ can be seen.

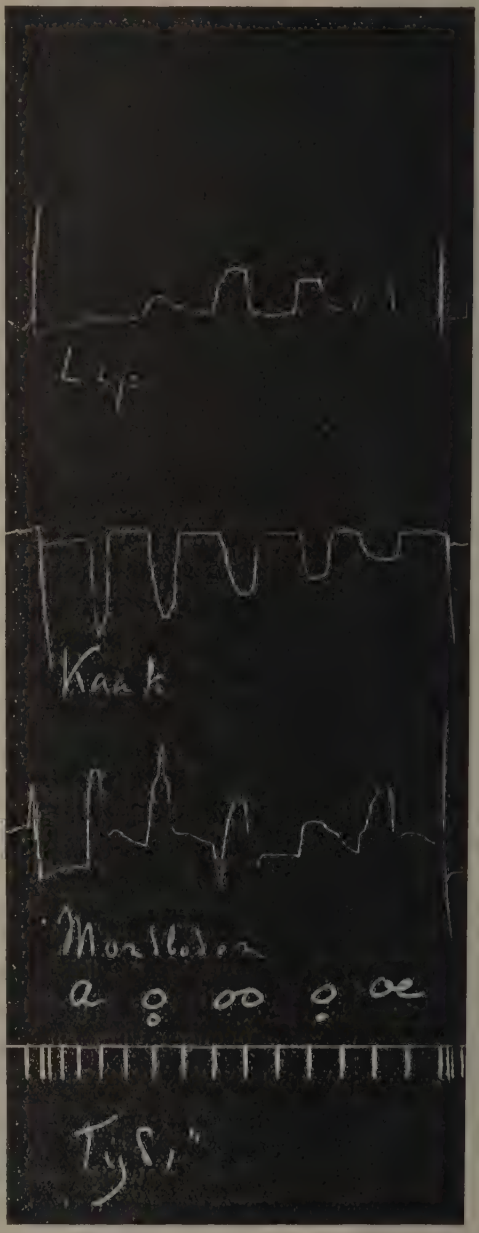

Fig. 4.

[1]) Leerboek der Physiologie II, p. 86.

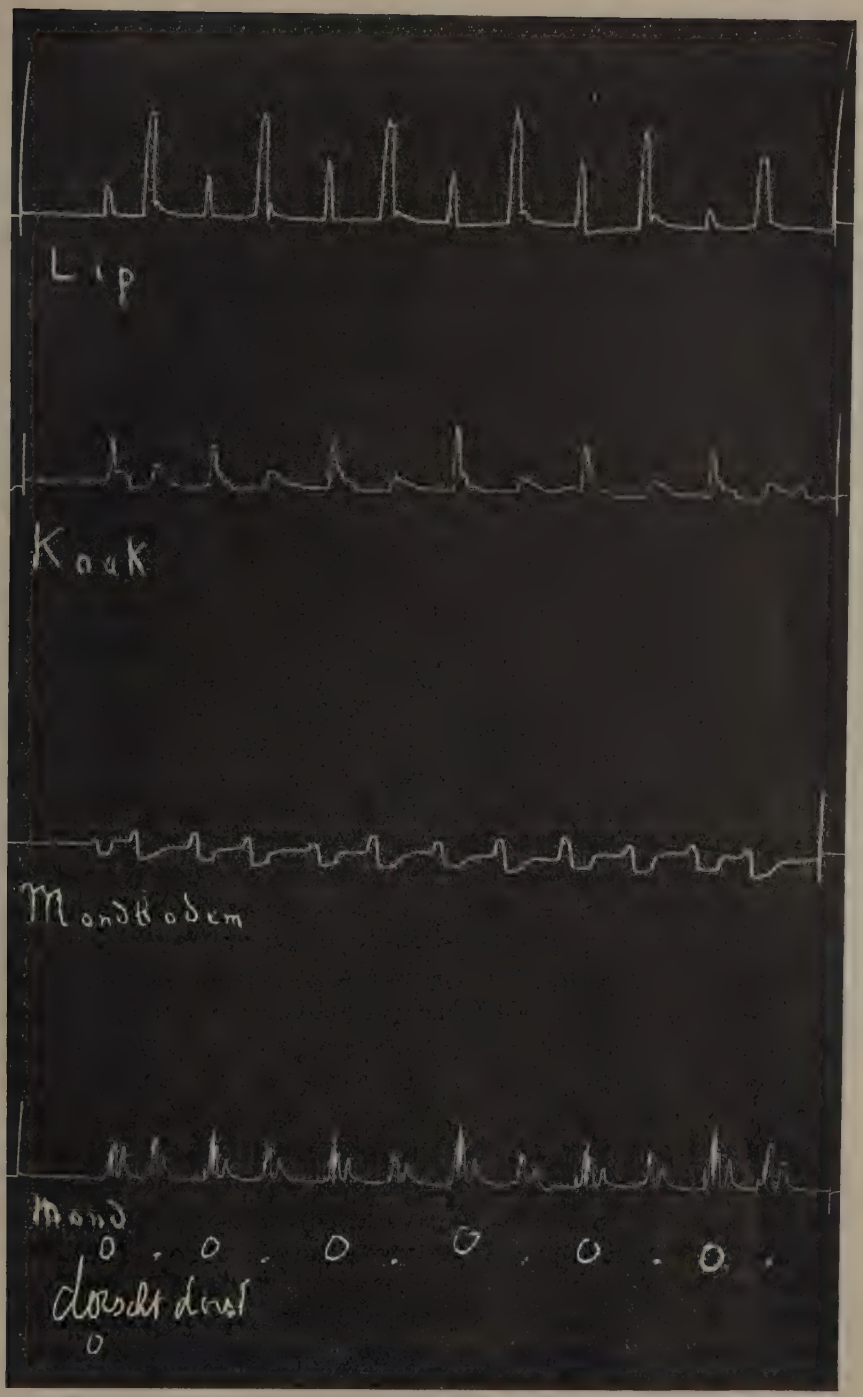

Fig. 5.

That the speech movements made to produce o̥ and o̩ as distinguished by the ear differ considerably, has been sufficiently proved in the above.

Fig. 6. Pp. v. d. S.

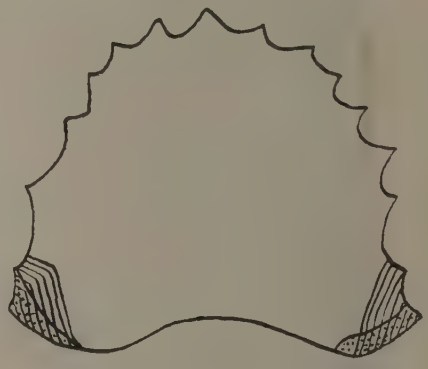

Fig. 6. Pp. R.

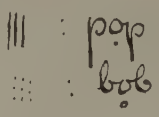

Fig. 6.

Experimental phonetic analysis of the sounds.

Also the characters of the sounds themselves proved to show a difference which could be easily recorded. In the first place the sounds can be easily registered on the kymographion. It need hardly be said that the tambour used for this purpose has to answer special requirements. This part of the inquiry was made under the guidance of Prof. Chlumsky. The tambour had the same shape as the recorder of a phonograph, the membrane was made of mica. An aluminium "mouthfunnel" after Rousselot was connected with this tambour by means of a wide rubber tube. Fig. 8 shows curves of the two sounds as registered in this manner. As a matter of fact the vibrations of a membrane like this are not large, owing to its stiffness; it is however partly due to this fact that we get curves which are thoroughly characteristic of the sounds recorded. So in our case there is a clear difference between the curve of o̥ and that of o̩.

The sounds can also be registered by means of a phonograph. A few monosyllabic words in which either of the two sounds occur according to the meaning (e.g. bo̥d and bo̩t) as well as the

sounds pronounced separately were recorded by means of an Edison phonograph (old type).

The difference between the sounds as recorded by the phonograph can be made much more illustrative and easily measurable by

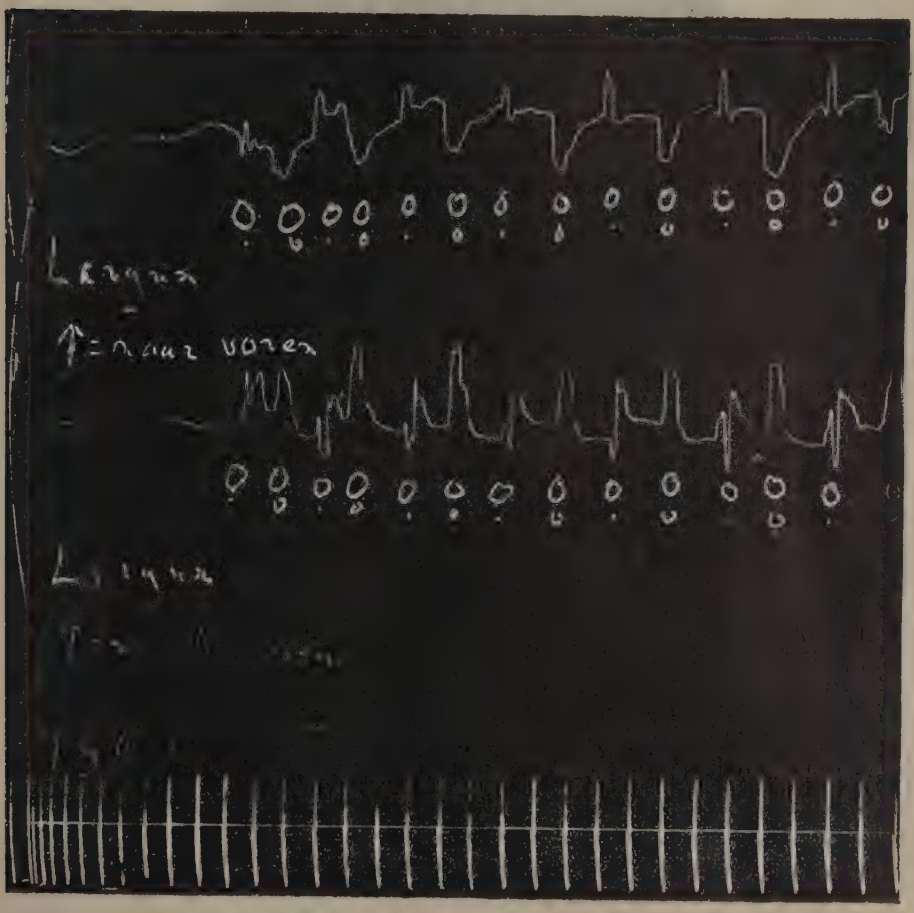

Fig. 7.

transforming the indentations of the wax cylinder into a curve on smoked paper. This is done by the apparatus constructed for this purpose by Lioret. A sapphire follows the groove of the phonographic cylinder; the movements made by the sapphire in doing so are transferred to a writing-lever, recording them on a rotating cylinder. As there is no such apparatus in Holland, as far as I know, also this part of the investigation was made at Prague under the guidance of Prof. Chlumský. The words „bọd" and „bọt" were again recorded phonographically, making use of the apparatus of Lioret. By means

of the same instrument these curves were subsequently magnified 300 times and registered on a smoked cylinder. Fig. 9 shows part

Fig. 8.

of these curves. The upper and lower curves represent the o̩-sound in "bo̩t", pronounced in a low voice in the former and loud in the latter. The curve in the middle gives o̞ cound in „bod" The difference between the two sounds is clearly revealed in this way and can be easily put into figures. The curve of o̞ is much the same as that which characterizes the aa-sound.

After a considerable and constant difference has thus been ascertained, it may be desirable to get an idea of the circumstances in which o̩ and o̞ occur in Dutch.

Fig. 9.

Linguistic remarks.

Every Dutch word of one syllable, containing o and also syllables with o, not occurring by themselves but with which influence from other syllables can be safely excluded (e.g. lom(mer)) were considered. Combinations of sounds that can be pronounced quite well, but are not found in the Dutch language, have been omitted.

As regards the influence by the several consonants, a few facts could be ascertained.

The most constant influence is that of following nasals. In this combination, namely, the o̜-sound occurs invariably. This can be easily comprehended, as the narrow mouth-opening and weak current of air passing through the mouth, promote the air current through the nose which follows.

Another influence is that of the lip-sounds; these promote the producing of the o̜-sound especially when preceding it. Also this becomes clear if we consider the narrowness of the mouth-opening. Guttural sounds like h, g, k, etc. are as a rule followed — and as far as they can be final, also preceded — by o̜. This also holds good, though in a less degree, for z, s, l, r, n and j; n of course only when preceding. D and t have no clearly manifested influence.

The r occupies a position of its own; its influence varies according to the way in which it is pronounced, which is different even with one and the same speaker at different times, its place of articulation varying between point of the tongue gums and root of the tongue-uvula. Taking this into consideration we can say that the advanced r promotes the o̜, the retracted r the o̜-sound, both whether preceding or following the vowel.

From this it appears that the adjoining consonants either promote the o̜ or the o̜. It should be borne in mind, however, that the only absolute influence is that of nasals as following sounds. The other influences only work to a limited degree. The fact that the influence of several sounds appears to be inconstant proves that there is at least one factor more playing a part. This becomes evident from the fact that several words, have either o̜ or o̜ according to their meaning, e.g.:

bo̜t (noun = flounder, bone; adjective = blunt)	& bo̜d (noun of bieden = to bid)
do̜l (adjective = mad)	& do̜l (noun, part of a rowing-boat = thowl)
do̜rst (noun = thurst)	& do̜rscht (from the verb dorschen = to thresh)

mọtje (dialect of moet-je = must you)	& mọtje (noun, dimunitive of mot = moth)
pọrt (from the verb porren = to stir)	& pọrt (noun from porto, oporto)
tọbbe (noun = tub)	& tọbben (verb = to worry)

It seems to me that the above may induce us to think of etymological influences. Words that have ọ in Dutch, usually occur in German with u, while those with ọ either have o, a, or au in German. I do not venture to judge about the value of this phenomenon. Other cognate languages as well may give indications. It may be wortwhile to make an etymological inquiry in this connection.

If etymological influences are ascertained indeed, we can imagine that they be inconsistent to a certain extent with the other influences described above. The word "pols", for instance, may be mentioned in this connection, because the pronunciation is wavering. It appears to me that this word is pronounced "pọls" by more careful speakers, while the majority say "pọls". Judging by its etymology the former pronunciation would be the right one; the latter may be easier because of the l that follows.

Summary.

There are in Dutch two short o-sounds that can be clearly distinguished both acoustically and phonetically (perhaps also etymologically).

Botany. — "*Ringing Experiments with variegated branches.*" By Prof. Th. Weevers. (Communicated by Prof. J. W. Moll.)

(Communicated at the meeting of September 29, 1923).

For a long time already the transport of carbohydrates and proteins in plants has been considered as a question that seemed fairly set at rest. Of late years, however, the problem has again been brought into prominence.

The well-known ringing experiments, notably the extensive observations made in this field by J. Hanstein [1]) had settled the belief that the organic matter was transported along the elements of the phloem. It was left undecided whether the elements of the cribral system (sieve-tubes and companion cells) or those of the parenchymatous phloemsystem (cambiform cells) play the principal part. Czapek's [2]) experiments favoured the first view, however, owing to the diametrically opposite conclusions of Deleano [3]) a decision was impossible at the time.

The primary and the secondary phloem was generally considered as the passage for the conduction of the organic products, which, being formed in the leaves, have to be conveyed to the growing points and the reserve-organs.

In accordance with Th. Hartig's [4]) conception it was, however, generally received that in the early spring, when the woody plants start new shoots, the organic matter finds its way from the reserve-stores to the shooting parts through the xylem. This hypothesis was based partly upon the results of Hartig's experiments with ringed plants and partly upon A. Fischer's [5]) observations regarding the occurrence of carbohydrates in the wood vessels. Researchers refrained from approaching the question as to how this happens in shooting herbaceous plants.

Now the above theory has latterly been impugned from various quarters.

[1]) J. Hanstein, Jahrb. f. wiss. Botanik, 1860.
[2]) Czapek, Jahrb. f. wiss. Botanik, 1897.
[3]) N. Deleano, Jahrb. f. wiss. Botanik, 1911.
[4]) Th. Hartig, Bot. Ztg., 1858.
[5]) A. Fischer, Jahrb. f. wiss. Botanik, 1890.

On the one side Otis Curtis[1]) made single and double ringing experiments and arrived at the conclusion that the transport of carbohydrates and proteins to the shooting parts may occur through the secondary phloem just as well as the transport in the opposite direction does, when the surplus of assimilates is removed from the place of formation. In my judgment, however, his view has not been sufficiently reinforced by indispensable quantitative examination.

On the other side it is Atkins[2]) and Dixon[3]) in England, and Luise Birch Hirschfeld[4]) in Germany who deny almost any significance to the phloem for the matter-transport. Their arguments consist in the main of indirect evidence. Atkins argues that the bleeding saps are more or less rich in carbohydrates not only in spring but also in other seasons. Luise Birch Hirchfeld and afterwards Dixon base their most cogent arguments upon their belief that an adequate transport of matter along the phloem can hardly be presumed. This difficulty had already been obviated by Hugo de Vries[5]), who made a quicker transport than the law of diffusion admits conceivable by assuming protoplasm-streams in the phloem-elements. Dixon, however, considers the impossibility of a transport of adequate capacity along the phloem as conclusive evidence for denying any significance to the phloem in this respect. Birch Hirschfeld is less positive in her assertion.

That, beside an ascending stream in the wood, there may also be a coinciding transport along it towards the bottom of the stem, may be concluded from various investigations i.a. the above-named by L. Birch Hirschfeld. Then the rate of transport can be much quicker than in the phloem and the capacity of the conducting channels can likewise be greater, as it is a fact that the phloem-production of cambium is invariably smaller than that of the xylem, while the generated phloem is obliterated much sooner.

This conception of Dixon's, however, does not square with the result of the ringing experiments of Hanstein, which result points indubitably to the stream of assimilates being stopped when the ringing wound is made deep enough to reach the cambium. Dixon therefore assumes the transport to pass through the youngest parts of the secondary xylem, which parts being located close to the cambium, are by him believed to be injured and thus rendered inactive by the ringing.

[1]) Otis F. Curtis, American Journal of Botany, 1920.
[2]) W. R. G. Atkins, Some recent researches in Plant Physiology, 1916.
[3]) H. H. Dixon, Pres. Address. Bot. Society, 1922.
[4]) L. Birch Hirschfeld, Jahrb. f. wiss. Botanik, 1920.
[5]) Hugo de Vries, Bot. Ztg., 1885.

To my knowledge this hypothesis has not yet been substantiated by experiments, so that it seems expedient to reconsider the question along what way the carbohydrates and the proteins are transported in plants.

The question can be approached from different sides; in this paper I will confine myself to a discussion of some experiments with ringed branches of variegated plants.

Similar experiments have been made repeatedly with green branches, but then the trouble is that after the buds have opened out, the younger parts above the ring begin to assimilate.

Stripping off the leaves or moving the plant to a dark space involves other difficulties; with variegated shoots it is much easier to state any supply of organic matter.

In consideration of DIXON's hypothesis due precautions should be used in the ringing and the protection of the injured part. A coating of melted butter of cocoa I deem more effectual than one of paraffin. It was applied to the wound at a temperature of 32°—33° C. and can hardly injure the exposed surface, as it does not penetrate into the intact cells.[1]) Moreover, it soon congeals and then affords sufficient protection against outside influences. The parts were then screened from immediate effect of the sun's rays in order to prevent melting.

We performed our experiments with variegated branches of Aesculus hippocastanum L. and Acer Negundo L. The former were derived from a stout specimen, whose green top provided the trunk with abundant food and from this trunk numerous yellow shoots had developed. In about 20 years these shoots attained a length of 1 M. and a thickness of 7—8 mm. in diameter. The specimen of Acer Negundo was provided at the top with green-white variegated leaves and developed from the main stem and side branches perfectly white shoots. In neither specimen did the yellow-white leaves contain any chlorophyl[2]). An iodine test pointed to the absence of starch.

In the spring experiments the branches were ringed (1—2 cm.) just before the buds began to open out and at a distance of 1—2 dm. below the end-bud.

Three series of experiments were always made at a time.

1st *series*: green shoots ringed all round.

2nd *series* variegated (yellow-white) shoots ringed all round.

3rd *series* variegated (yellow-white) shoots partially ringed, viz. so as to leave as trip of bark as a connecting link, 2—4 mm. in breadth.

[1]) R. H. SCHIMDT, Flora Bd. 74, 1891.

[2]) Guard-cells of the stomata excepted.

After rather more than a week a contrast was noticeable between the green and the partially ringed variegated shoots on the one side and the completely ringed variegated shoots on the other. The first two (1st and 3rd series) continued growing normally. The third (2nd series) lagged behind and died off after 2 or 3 weeks, the leaves having previously shrivelled and dried up.

Fig. 1.

That ringing in itself did not injure the plant appeared distinctly from the results of the first and the third series. (See the photos): from left to right we see first 4 completely ringed yellow branches, some brown and dead, others small but still living; the next following are two completely ringed green ones and lastly to the right two partially ringed. The last four have developed normally.

It is clear that with the completely ringed green shoot the supply of water is normal; why then does the completely ringed yellow branch die off under symptoms that point to a deficiency of water?

The reason is obvious. In consequence of too little osmotic pressure the absorptive power of the tissues is too low as compared with that of the other parts.

The researches by Dixon and Atkins [1]) on the determination of the osmotic pressure by lowering the freezing point of the expressed

[1]) Notes Botanical School. Trinity College Dublin, 1912.

sap, clearly show how the osmotic value of the leaf-cells increases with the possibility of assimilation.

Now I endeavoured to determine the suction force by URSPRUNG's [1]) method but the subject appeared to be difficult to experiment on.

A quantitative determination gave in the green leaves of Aesculus an amount of reducing sugars of 3 %[2]), in the variegated (yellow) leaves 1 %, in the ringed variegated (yellow) branches only traces. In general also the amount of extractable salts is trifling; in green and variegated leaves 0,9 % of the fresh weight [3]). SPRECHER finds in yellow varieties lower osmotic values for the cell sap than in the green specimens [4]).

True, the variegated leaves of the ringed branches of Aesculus contain from 18 to 20 % protein and 5 % dextrin (calculated at dryweight) but the influence of these amounts on the osmotic pressure is nothing to speak of. Yet this does not explain all, for in the variegated completely ringed shoots wood and bark above the ringing appeared to contain still a fair amount of starch (6 % of the dryweight, against 9 % in the partially ringed branch), while the leaves were already shrivelling.

Why this starch is not converted into sugar and why, when transported to the leaves, it does not raise the osmotic pressure has not yet been explained.

However this may be, the partially ringed variegated branches do not die off. It appears, then, that there the supply is not cut off and that consequently the young parts are provided with the nutriment that in the green ringed branches is produced by assimilation.

According to HANSTEIN the organic products are conveyed along the bridge of bark, but if this is the case, we must relinquish HARTIG's hypothesis that the transport is effected along the xylem while the branches are budding.

OTIS CURTIS (l.c.) does so and was led by his ringing experiments to regard the phloem exclusively as the path, along which the saps

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[1]) URSPRUNG, Ber. d. d. bot. Ges., 1918.

[2]) Strictly speaking 2 % and 1 % reducing sugars derived from glucosids (calculated at dry-weight).

[3]) The starch determinations were performed by putting the pulverized material immersed in water for 3 hours into an autoclave at 4 atm., and by subsequently boiling the aequeous extract with diluted hydrochloric acid during 60 minutes.

Plasmolytic experiments are objectionable on account of the osmotic pressure in the various cells being unequal. Still, a 10 % saccharose solution plasmolyzes the variegated Aesculus-leaves, not however the green ones.

[4]) A. SPRECHER. Rev. Gen. Bot. 1921.

are transported. From Dixon's point of view, however, it might be objected that in Curtis's experiments the peripheral woodlayers were injured and thereby the transport along the peripheral xyleem had been suspended indirectly.

This objection can hardly be raised against the above experiments, in which a coating of butter of cocoa was spread on the injured part.

Fig. 2.

Moreover, another series of ringing experiments was carried out. In these experiments the ringing was performed as much as possible aseptically by previously washing the branch bark with 96 % alcohol and then peeling it off aseptically down to the cambium. Subsequently the decorticated surface was covered with sterilized absorbent cotton wool saturated with water; finally the whole was wrapped up with wax taffeta.

These experiments were carried out mid-June in the same way as the others described above, and yielded after four weeks an unequivocal result in connection with the midsummer growth which was very abundant, especially in Aesculus.

With the normal yellow variegated shoots the formation of midsummer growth occurred at the top of the branch and the yellow young leaves contrasted sharply with the others, which had been damaged by the high wind and browned by the sun. (See photo).

It appears then, that here also the yellow leaves suffer under a deficiency of suction force, and under circumstances brought about by stronger evaporation are sooner destroyed than the green ones, although the latter evaporate comparatively more intensely.

With partially ringed variegated shoots the midsummer growth occurred also at the top. With completely ringed specimens, however, it appeared *below* the surface of the wound from lateral or dormant buds. (See photo). This occurred as well when the surface of the wound was covered with butter of cocoa, as when it was dressed with a water-bandage.

The check to the food-supply is apparently as great with Aesculus as with Acer Negundo, in spite of the greatest precaution used in cutting the ring. It follows, then, that the experiments do not yield any evidence whatever, to lend support to DIXON's theory. They rather go against it.

Still conclusive evidence to disprove DIXON's theory cannot be brought forward by this procedure, since in spite of all due precaution the peripheral wood may be prevented by the ringing from performing its function, as far as the transport of the organic products is concerned.

With regard to other inquiries, whose results tell strongly against DIXON's theory, we first of all have to think of HANSTEIN's experiments (l.c.) on the root-growth of ringed branches in water culture.

HANSTEIN finds that detached branches placed in water send out roots chiefly at the basal extremity of the stem, which VÖCHTING ascribes to the polarity of the parts. Leafless branches when ringed develop a large number of roots just above the wound; whether and to what number they will grow at the bottom of the branch, depends on the distance between that extremity and the ringing.

HANSTEIN ascribed this to the check to the transport of nutriment consequent on the removal of the phloem, and established, indeed, in such circumstances a distinct difference in the root-growth, between dicotyledonous plants with an anomalous stem-structure and those with a normal stem-structure, in which the stem derived its thickness form a ring of collateral vascular bundles. With the former the transport of carbohydrates and proteins is believed to be only partially checked. This is ascribed to the fact that the vascular bundles are contained within the xylem (as with Piperaceae and Nyctaginacea) or (as in the case of Apocynacea and some Solanacea) to the fact that there are originally bicollateral vascular bundles or rather medullary phloem strands and consequently phloem remains also within the secondary xylem. Owing to this HANSTEIN stated

in this case only a very slight influence of the ringing upon the root-growth.

This evidently does not fit in with DIXON's view; if the transport is effected along the peripheral parts of the xylem, ringing must in these plants have the same effect. It struck me, therefore, that it would be worth while to repeat some of HANSTEIN's experiments. The Solanacea Cestrum aurantiacum proved to be an unsuitable subject since detached branches sent out roots very sparingly in water culture, but Nerium Oleander yielded quite satisfactory results: all the twelve cuttings presented an aspect, quite in harmony with HANSTEIN's description. The root-growth may be somewhat more abundant above the wound, but the behaviour is quite different from e. g. that with Salix and Cornus spec. In these the roots appear almost exclusively above the wound, unless the stem-piece below it be very long, and the once formed roots are even destroyed when the bark above them is stripped off.

Provisionally all this tells very strongly against the validity of DIXON's conception of a transport of the carbohydrates and the proteins along the peripheral xylem.

If the above-discussed experiments with variegated shoots could also be made with variegated Oleanders, the medullary phloem of these plants would probably cause a quite different result from that yielded by Aesculus and Acer. But unfortunately variegated Oleanders I had not at my disposal, so that now I made a trial with ringed, normal shoots, which, while still attached to the plant, were wrapped up in black paper. The result was rather conclusive. Although some leaves had fallen off, the shoots themselves were still alive ten weeks after the ringing and had increased in length.

We see, therefore, that not only in the formation of the roots of branches in water-culture but also in the budding and the growth of Oleander Aesculus and Acer Negundo in spring, the results of our experiments with ringed branches imply a transport along the phloem.

In a subsequent publication I intend to discuss the question whether the capacity of these paths is sufficient.

For the present the above observations on Aesculus and Acer Negundo, where the detached branches did not bleed, are not applicable to the cases in which this bleeding is so copious, and as with Betula alba the highly sacchariferous sap is exuding directly after the ringing [1]).

[1]) The cases described by MOLISCH. (Bot. Ztg. 1902) as wound-reaction with local bleeding pressure, are of quite a different nature; then the bleeding pressure manifests itself only after days or weeks.

Physiology. — "*Determination of the Power of the Accommodation-Muscle*". By Prof. J. VAN DER HOEVE and H. J. FLIERINGA.

(Communicated at the meeting of September 29, 1923).

The action of the accommodation muscle, the M. Ciliaris, makes itself apparent to us by the increase of refraction of the lens, the so-called accomodation of the eye.

There are still many obscure points in the subject of accommodation; for instance, it is still entirely unknown to us what relation exists between the contraction of the accommodation-muscle and the increase in refraction of the lens.

A few ophthalmo-physiologists are of opinion that contraction of the accommodation-muscle increases the tension in the ligament of the lens, the Zonula Zinii, while most of them assume, with HELMHOLTZ, that contraction of the ciliary muscle causes a relaxation of the Zonula Zinii, so that opportunity is given to the lens to curve according to its elasticity. When, through increase of age, the elasticity disappears, contraction of the ciliary muscle does not assert itself by increase of refraction of the lens.

Even if one assumed the last theory, one meets with many unsolved questions, e g.:

a. Is the strongest possible contraction of the accommodation muscle necessary to obtain the greatest possible accommodation? DONDERS and LANDOLT assumed this and find still followers in these days, amongst others CLARKE and DUANE.

FUCHS, HESS and others, on the contrary, are of opinion that the accommodation muscle can contract far more strongly than is necessary to obtain a maximal accommodation.

FUCHS expresses this in the following way: the accommodation-muscle can first contract so far that the lens can follow its elasticity completely, resulting in a maximal accommodation; the eye is then focussed on a point, which is determined by a physical property, viz. the elasticity of the lens. FUCHS therefore calls this point the "physical near point". Now he muscles can contract considerably more, so that the Zonula hangs entirely relaxed and the lens could if only its elasticity were unlimited, increase its refraction considerably, allowing the eye to focus on a point, lying still closer by, and

determined by a physiological property, viz. the power of contraction of its accommodation-muscle: the physiologic near point.

Hess says that it is almost generally assumed that every increase in lens-fraction of one dioptrie exacts an equal increase of the contraction of the ciliary muscle.

Although this simple relation is not self-evident, considering the complicatedness of the accommodation-process, we will accept it for a moment, in order to try and prove it, taking as unit of contraction of the ciliary muscle, the contraction necessary to bring the accommodation from 0 to 1 dioptrie, which unit we can call „myodioptrie".

If Hess' unproved supposition is correct, one will need a contraction of 10 myodioptries in order to be able to accommodate 10 dioptries. We can now also express the total power of the accommodation-muscle in myodioptries, for in an emmetropic person this will be the reciprocal value of the distance of the physiological near point, or otherwise expressed, it will be equal to the number of dioptries one could accommodate if the lens had an unlimited elasticity.

So we stand here before the following two questions:

b. Is the myodioptrie for one person a fixed unit? That is to say: is a contraction of the accommodation-muscle of one myodioptrie necessary for every accommodation-increase of one dioptrie?

c. How great is the power of the accommodation muscle expressed in myodioptries?

Other questions which rise before us are the following:

d. Is it possible to detect the very slightest paresis of the accommodation muscle?

e. Is it possible to make curves of the paralysing influence certain substances exert on the accommodation muscle?

Up to now we were accustomed to determine the action of the accommodation-muscle by finding the nearest point.

Let us now suppose a person (fig. 1) who can accommodate 10 dioptries, and who possesses a power of the ciliary muscle amounting to 24 myodioptries, then the accommodationmuscle can be more than half paralysed, while the nearest point need not have changed its place. In this way we only notice the possible presence of a paralysis of the accommodation muscle, when it is far advanced.

In consequence we know little about the paralysing action of certain substances which only slightly affect the accommodation muscle, even about those substances which we use daily, such as cocaine. We find the most divergent communications in the literature about the paralytic action of cocaine on accommodation.

Some writers assert that it does not act at all on the accommo-

dationmuscle, others say that it acts very strongly, and a third group states that it does work, but only slightly.

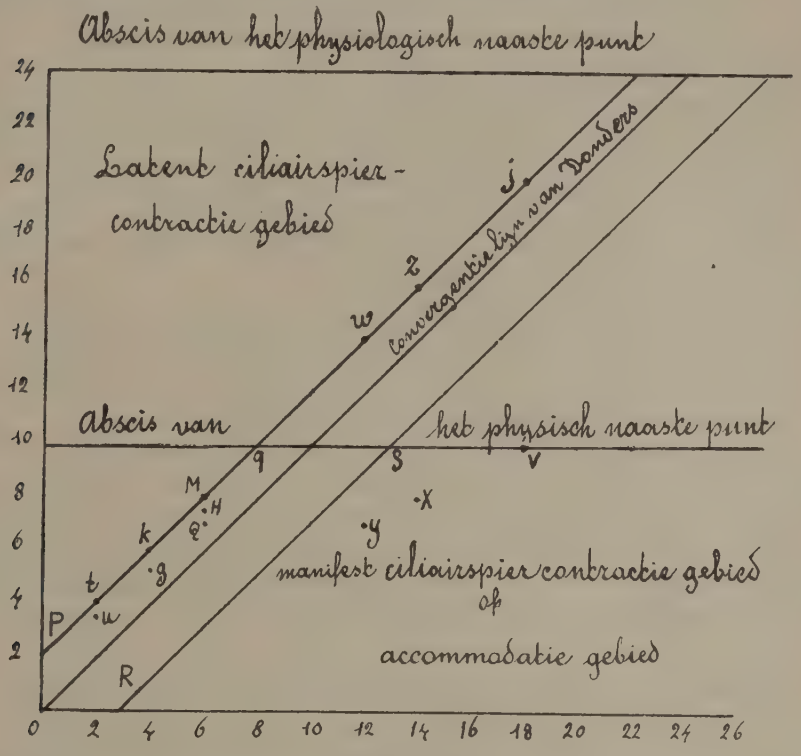

Fig. 1.

Abscis van het physiologisch naaste punt	=	Abscis of the physiologic near point.
Latent ciliairspiercontractie-gebied	=	Area of latent ciliary muscle contraction.
Convergentielijn van Donders	=	Donders' convergenceline.
Abscis van het physisch naaste punt	=	Abscis of the physical near point.
Manifest ciliairspiercontractiegebied of accommodatiegebied	=	Area of manifest ciliary muscle contraction.

Dr. FLIERINGA and I have tried to solve the foregoing questions by a minute study of the relative accommodation.

By relative accommodation we understand the accommodation at a certain given degree of convergence. A certain connection, probably congenital, exists between accommodation and convergence; if a normal emmetropic person wishes to fix his eyes on an object, he must converge as many metreangles as he accommodates dioptries.

If, in Fig. 1, we plot out the myodioptries and dioptries on the ordinate, and the metreangles on the abscis, then we can draw a line through all the points for which accommodation and convergency are alike; if, in our scheme, we take the linear measure for

metre angle and dioptrie the same, then this line divides the right angle between ordinate and abscis exactly in two equal parts. This line, which unites all the points denoting an equal number of metreangles for convergence as dioptries for accommodation, is called: "DONDERS' Convergence-line".

If the relation between accommodation and convergence was absolute and unfringible, then a normal person would only be able to see the points of the convergence-line sharp and single at the same time, and no other points; every person with an *abnormal* refraction or a heterophoria would not be able to see one single point sharp and single at the same time.

Luckily the connection between accommodation and convergence is more or less a loose one, so that at every convergence the acommodation can, to a certain degree, be made stronger or slighter than coincides with the degree of convergence.

If one converges 6 metreangles, then an accommodation of 6 dioptries coincides with this, an accommodation, which one can raise f.i. to 8 dioptries, or decrease to 3 dioptries. This interval between 3 and 8 dioptries is called the relative accommodation for a convergence of 6 metreangles; the interval from 6 to 8 dioptries is called the positive, from 6 to 3 dioptries the negative relative amplitude of accommodation.

The relative amplitude of accommodation differs a great deal in each individual case and can be increased to a certain degree by long practice. It is not necessary that the negative and positive part of the relative accommodation are alike.

One can determine the relative accommodation for all points in the area of manifest contraction of the ciliary muscle and connect the relative near and far points to get the lines of the relative near and for points.

According to HESS the relative accommodation is the same at every convergence, so that for every normal person the lines of the relative near and relative far points run parallel to DONDERS' Convergence-line. (See fig. 1: pq and R.S.)

HESS is of opinion that one can continue these lines in the area of latent ciliary muscle contraction, but could not prove this, as no measuring could be done in the "latent" area.

The next question therefore is:

f. How do the lines of the relative near and far points run in the area of latent ciliary muscle contraction?

Our reasoning is as follows: if the supposed individual of fig. 1 converges 6 metreangles, the unparalysed ciliary muscle can contract

through the stimulus of this convergence, and with the utmost exertion, 8 myodioptries, and can therefore accommodate at M: if however, the muscle is paralysed in the slightest degree, it will contract less strongly through this same stimulus, e. g. only $7\frac{1}{2}$ myodioptries, and will therefore accommodate at H.

By determining the relative accommodation, we can therefore detect the slightest paralysis of the muscle in an individual of whom the accommodation-figure is known (question *d*).

To see if the myodioptrie is a constant value, we paralyse the muscle to a certain degree, for instance so that on converging 6 metreangles, accommodation is only possible as far as Q; the muscle then has an action of 7 instead of 8 myodioptries; if all myodioptries are of equal value, then the muscle only possesses 7/8 of its power and is for 1/8th paralysed. We control this by measuring the relative accommodation and determining the degree of paralysis for the same paralysis and other convergencies too.

If one constantly finds the same degree of paralysis, so that on converging 4 metreangles an accommodation only takes place up to $g = 5\frac{1}{2}$ myodioptries, instead of 6; and on converging 3 metreangles, there is only an accommodation to $U = 3\frac{1}{2}$ myodioptries instead of 4; then the paralysis appears to be constantly 1/8. One can control this with as many degrees of paralysis and convergencies as one wishes, so that question *b*, whether the myodioptrie is a constant value in one particular person, can be definitely solved.

To determine the course of the lines of the relative near and far points in the latent area, one paralyses the accommodationmuscle to a certain degree, say the half, so that one finds by convergence of two metreangles (in fig. 1) a greatest accommodation of 2 D., instead of 4 D.; 3 D. instead of 6 D., on converging 4 m. a.; and 5 D., instead of 10 D., with a convergence of 8 m. d.; if, now, on converging 12 metreangles a greatest accommodation of 7 D. is reached (to Y in fig. 1), then one may say that the half-paralysed muscle contracts 7 myodioptries with this stimulus; the normal muscle would therefore have reacted with $2 \times 7 = 14$ myodioptries, so that the relative near point with a convergence of 12 metreangles would lie at W., on the ordinate of 12 and the abscis of 14.

If, with a convergence of 14 metreangles, one finds a greatest accommodation of 8 D., (to X in fig. 1), then the healthy muscle would be able to contract 16 myodioptries, in answer to this stimulus. thus fixing the point Z on the abscis of 16 and the ordinate of 14.

If with a convergence of 18 metreangles one finds an accommodation of 10 dioptries, then point j on the abscis of 20 myodioptries is

determined. In this manner one can determine in the latent area as many points of the line of relative near points as one wishes, with different convergencies and different degrees of paralysis, and so plot out the entire line.

The course of the line of relative far points in the area of latent ciliary muscle contraction, is determined in the same manner, so that question *f.* is solved.

We determine the strength of the accommodation muscle in the following manner:

When the area of relative accommodation has been completely ascertained, the muscle is paralysed. Supposing that in the individual of fig. 1, the accommodation muscle is paralysed for $^1/_4$th part; the absolute near point is now determined; if this still lies at a distance of 10 cm., then one can say that $^1/_4$ of the muscle-power produces a contraction of at least 10 myodioptries, the total muscle power is therefore at least $4/3 \times 10 = 13\ 1/3$ myodioptries.

If the paralysis is 1/3 while the accommodation remains 10 D., then firstly one may consider question *a.* as answered; for a partially paralysed muscle can evidently give the greatest possible accommodation, so the strongest possible contraction is not necessary, and secondly 2/3 of the muscle-power produces a contraction of at least 10 myodioptries, the muscle-power is therefore at least $3/2 \times 10 = 15$ myodioptries.

If again with a paralysis of $\frac{1}{2}$ an accommodation of 10 D. is reached then the power is at least 20 myodioptries. But if, on paralysing the muscle for one third of its power, only 8 D. accommodation is reached, then the power is $3 \cdot 8 = 24$ myodioptries.

Control is obtained by further paralysis; if, after paralysing three quarters of the power, 6 D. accommodation is reached, then the total power is $4 \times 6 = 24$ myodioptries; if there is still an accommodation of 4 D., after the muscle has been paralysed to $^1/_6$, then the power is $6 \times 4 = 24$ myodioptries, etc., so that the result obtained can be controlled by as many observations as one wishes.

If all the values obtained coincide sufficiently, then one has not only determined the muscular power, but has also proved that the myodioptrie has a constant value and that the method must be correct, otherwise the values could not constantly be found to coincide.

A curve of the paralysing action of a substance can be obtained by first determining the total power of the muscle in a certain individual, then dropping the paralysing substance in the eye, and determining the power again, at regular intervals, the results being plotted out in a scheme.

To this purpose we note (fig. 2) the time in minutes on the abscis, the muscle-power in myodioptries on the ordinate.

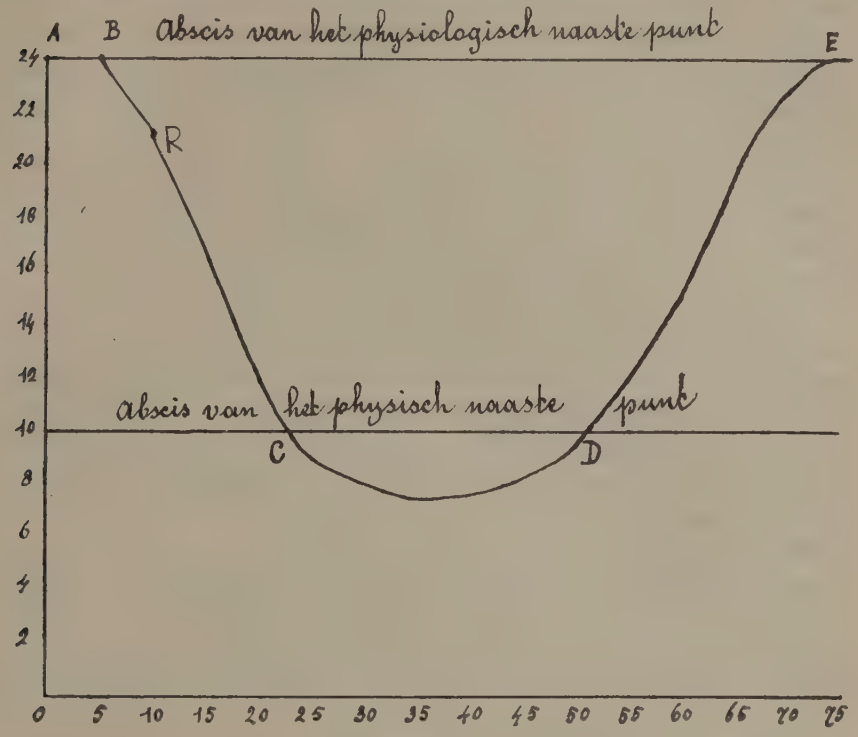

Fig. 2.

Abscis van het physiologische naaste punt = Abscis of the physiologic near point.
Abscis van het physisch naaste punt = Abscis of the physical near point.

Supposing the power, at the beginning of the examination, to be 24 myodioptries, when after 5 minutes there is no sign of paralysis, then one notes point B on the abscis of 24 and the ordinate of 5; if, after 10 minutes, the muscle is paralysed for $^1/_8{}^{th}$ part, then the power is still 21 myodioptries, and one has found a point R on the abscis of 21 and the ordinate of 10. Continuing in this manner, and continually determining the degree of paralysis of the muscle, one can find and plot out the entire paralysis-curve A. B. C. D. E. This examination gives an excellent control of the correctness of the method; for as soon as the curve surpasses the abscis of the physical near point, we can also directly find the degree of paralysis by determining the absolute near point. The part C.D. of the curve can therefore be ascertained in two entirely different ways.

If these two give entirely the same result, or if they agree

sufficiently (taking into consideration the possible errors of the method) then we may look upon this as a proof of the correctness of the method.

We have determined the "accommodation-figures" for a couple of persons, aged respectively 31 and 24 years, and have examined the paralysing action of cocaine and homatropine on the ciliary muscle.

One sees from our curves that the result is such that we feel justified in concluding that the method is good. In one patient we found a power of the ciliary muscle amounting to about 24 myodioptries; in the other 20 myodioptries.

It appeared that total contraction of the ciliary muscle is not necessary to obtain the greatest possible accommodation; that the myodioptrie has a constant value for each of these two persons, that the lines of the relative near and far points in the area of latent ciliary muscle contraction, run parallel to each other and to the convergence line of DONDERS, and that it is possible in persons whose "accommodation-figures" are known, to detect even the slightest decrease in power of the ciliary muscle.

Cocaine has on the accommodation-muscle a cumulative paralysing action, which shows considerable individual difference; it is therefore not at all surprising that one comes across such different reports of its action in the literature; as the possibility of detecting this action was dependent on:

the number of times cocaine is dropped in the eyes; the age of the observer; individual peculiarities; the duration of the observations and from the intervals between the observations.

One can draw still more conclusions from the results obtained, with regard to the influence of heterophoria, condition of refraction, etc. on the "accommodation-figures", and of the influence, which feebleness of the ciliarymuscle has on the power to do our work at short distance.

My only object at present, however, was to draw attention to the fact that the method of examining the relative accommodation enables us to widen our insight into the accommodation, and makes it possible to examine the influence of different substances on the accommodation muscle.

It is a pity that the method itself is so difficult to master, that it will never become a method for clinical examination in the hands of many, but will have to be limited to the laboratory work of a few.

Physiology. — *"On the Influence of the vagi on the frequency of the action currents of the Diaphragm during its respiratory Movements."* [1]) By Dr. J. G. Dusser de Barenne and Dr. J. B. Zwaardemaker. (Communicated by Prof. H. Zwaardemaker.)

(Communicated at the meeting of September 29, 1923).

In a previous paper one of us [2]) was able to show that the frequency of the action currents of the striped muscles, as they occur in the cerebrate rigidity of the cat and in the voluntary contraction in man, undergoes a distinct diminution after elimination of the proprioceptive impulses, originating in the muscles during their contraction. The elimination of these proprioceptive impulses was produced by section of the posterior roots in the animal and by local intramuscular injection of novocain in the human individual.

We then investigated if this experimental fact could also be established in other innervations and first of all in the diaphragm. We will not dwell here on this investigation which gave us similar results for this muscle as in the researches mentioned above. But in the course of these investigations on the frequency of the action currents of the diaphragm, we got results which gave rise to the supposition that perhaps the vagi might have an influence on the action currents of this muscle during its respiratory contractions.

We, therefore, had to investigate this problem separately and propose to deal in this paper with the obtained results. The question to be answered, was therefore the following: Which is the frequency of the action currents of the diaphragm during its respiratory contractions *before* and *after* elimination of both vagi.

At first we made use of the cat; later on, when we had already obtained a definite answer to our question, we did another set of experiments on the rabbit and could show that also in this animal

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[1]) A preliminary communication of this paper was made at the XI[th] International Physiological Congress at Edinburgh, 25th July 1923.

[2]) J. G. Dusser de Barenne, Untersuchungen über die Aktionsströme der quergestreiften Muskulatur bei der Enthirnungsstarre der Katze und der Willkürinnervation des Menschen. Skandin. Arch. f. Physiol., 1923, Vol. XLIII. (Festschrift für R. Tigerstedt), S. 107.

he vagi have the same influence on the action currents of the diaphragm, as found in the cat, this influence in the rabbit being even much more distinct than in the cat.

Anaesthesia of the animal by subcutaneous injection of urethane (ca. 1 gr. pro KG. of body-weight). By means of artificial heating we tried to keep the body temperature of the animal as constant as possible. Incision of the abdominal wall in the linea alba, of about 3 cm., beginning directly caudally of the ensiform cartilage. This processus was kept in upright position by fixing it with a forceps to a support, which was isolated electrically from the table on which the animal was lying. Then we isolated as carefully as possible one of the anterior slips of the diaphragm and put a small piece of celluloid under it, so as to insulate this part of the muscle as well as we could from the other parts of the diaphragm and its surroundings. In this slip were hooked two very small hooks at a distance from each other of about 1—1,5 cm., which served as electrodes, through which the action currents of the muscle were lead off to the string galvanometer (large pattern of EDELMANN). In our earlier experiments these hooks were of copper and therefore polarisable electrodes, in our later experiments we made use of similar shaped silver hooks, galvanoplastically coated with a layer of silver chloride; these electrodes were non-polarisable. As was to be expected we could not find that the use of these different electrodes gave rise to any appreciable difference in our curves, because it cannot be expected that the polarization of the copper hooks has a distinct influence on the weak, frequent and alternating action currents of the muscle. The hooks were connected with very thin copper wires to the thicker wires leading to the galvanometer, so that the movements of the muscle could be followed quite freely by the electrodes and connecting wires. By closing the opening in the abdominal wall with a pad of dry cottonwool loss of heat of the muscle and other disturbing influences were prevented.

The respiratory movements of the diaphragm were reproduced on a kymograph with blackened paper and underneath these tracings we marked electromagnetically during which part of the pneumogram the action currents were registered. The table with the animal was carefully insulated by putting it on large blocks of paraffine.

After these preliminaries we first took the action currents of the diaphragm during normal inspiration, i.e. before the elimination of the vagi. Then both these nerves were carefully prepared at the neck and eliminated without excitation, either through local anaesthesia with ether vapour or through local application on the nerves of a 1 % solution of novocain. When the elimination of the vagi established itself by a change of the respiratory movements of the animal, we again registered the action currents of the insulated anterior slip of the diaphragm. We might draw attention to the fact that by a special devise it was possible to take our electrophysiological records in every desired phase of the respiratory contraction of the muscle.

In all our experiments in which the elimination of both vagi is followed by a distinct change of the mechanical type of respiration, we could establish the fact that the elimination of the nervous impulses *gives rise to a distinct augmentation of the frequency of the*

action currents of the diaphragm during its inspiratory contractions. Only in those few cases, already known to ROSENTHAL, in which the respiratory movements remain nearly unaltered, could we find but a small augmentation. But even in these experiments an augmentation of the frequency was to be seen, though slight. Until now we have not yet met with an experimental result, pointing in an opposite direction, i.e. a diminution of the frequency of the action currents of the diaphragm after elimination of the vagi.

First of all we will give some curves as evidence of our statement.

fig. 1*a*

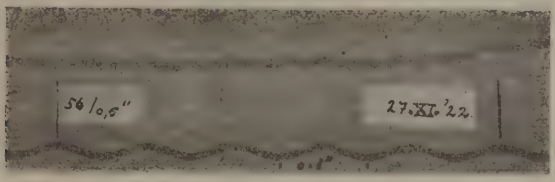
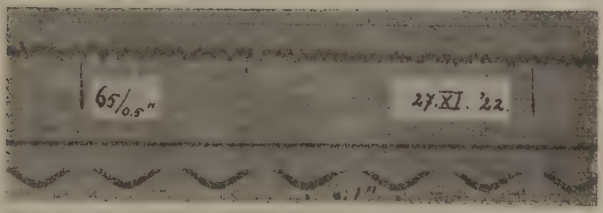


fig. 1*b*



Cat. Experiment of the 27th Febr. 1923. Fig. 1*a* action currents of the diaphragm *before*, fig. 1*b* *after* elimination of the vagi. Time 0.1 sec. On the original photographs in 1*a* 56, in 1*b* 65 action currents could be counted during the marked 0 5". So the frequency was 112 before, and 130 per sec. after the elimination of the vagi. (The date in these figures is wrong.)

fig. 2*a*

fig. 2*b*

Cat. Experiment of the 19th Dec. 1922. As foregoing figure. Frequency in 2*a* 98 per sec., in 2*b* 120 per sec.

Figures 1*a* and 1*b* show, although unfortunately not quite so distinct as the original photographs, that the frequency of the action currents before and after elimination of the vagi is **112** resp. **130** per second. In fig. 2*a* and 2*b* these numbers are **98** and **120** respective.

The results of our **8** experiments on the cat in the order in which they were performed, are given below in the table. In experiment IV only a slight augmentation of the frequency was found; in this animal the change of the pneumogram of the diaphragm after the anaesthesia of the vagi was not very distinct.

TABLE

of the frequency of the action currents of the diaphragm in the cat.

Number of experiment	Before the elimination of the nervi vagi.	After the elimination of the nervi vagi.	% augmentation.
I.	98	120	22.45
II.	118	130	10.17
III.	102	117	14.7
IV.	95	101	no distinct change in the pneumogram of the diaphragm
V.	118	130	10.17
VI.	112	130	16.07
VII.	113	132	16.81
VIII.	119	133	11.76

On the rabbit **6** experiments were made; in all of which an augmentation of the frequency of the action currents after elimination of the vagi was also found, for the most part still more evident than in the cat. In one of the rabbits this augmentation was even 40 %.

We will now try to answer the question, how one has to look at this experimental fact.

As is already long known the effect of double vagotomia, either by cutting or by local anaesthesia of the nerves, is that the respiratory movements become less frequent and are increased in amplitude. We will for the present confine ourselves to this last point. The muscles which perform inspiration and in the first place the most important, the diaphragm, contract more vigorously, after the elimin-

ation of the vagi. One might consider the most plausible explanation of our experimental fact to be the following, viz.: that this stronger contraction of the muscle might show itself in an augmentation of the frequency of its action currents. This explanation however is not consistent. First of all we know the fact already ascertained by PIPER, that the frequency of the action currents in voluntary contraction of human muscles remains unaltered under various strengths of contraction, a fact which one of us (D. DE B.) lately confirmed. But it might be argued, that this fact, though it may be true with regard to voluntary contraction of the human muscle, might not apply to the diaphragm of the rabbit. We, therefore, tried to get direct experimental evidence on this point by inducing a deepening of the inspiratory movements with other methods, f.i. by letting the animal breath an atmosphere rich in CO_2 or by closing the trachea during a few seconds. It was found that the deepening of the inspiration which follows these procedures is *not* accompanied by an augmentation of the frequency of the action currents of the diaphragm. We could establish this in many experiments; only in one of them we found that after breathing a CO_2-atmosphere there was also an augmentation of the frequency of the action currents. In this experiment we had already performed a local ether anaesthesia of both vagi; it might be possible that the nerves were still functionally slightly damaged; anyhow in all our other experiments, in which the increase of inspiration through CO_2-breathing preceded the vagotomia, we never found an augmentation by CO_2.

Only one objection must still be taken into account.

One of the other results of the elimination of the vagi is an acceleration of the heart. In our experiments, in which the anterior slip of the diaphragm was not detached from the ensiform cartilage for the sake of leaving the muscle in as normal a condition as possible, we generally also found in our curves of the action currents traces of the electrocardiograms of the animal, especially in the cat, where the insulation of the anterior slip of the diaphragm is much more difficult than in the rabbit. These electrocardiograms present themselves under these circumstances as simple, diphasic action currents, which look very much like the action currents of the diaphragm itself and often cannot be distinguished from them. So, when one counts all the peaks during 0.5 a second, as we always did, a few of these electrocardiograms are always included. The objection might now be made that after the vagotomia through the acceleration of the heart, the number of electrocardiograms is aug-

mented, and that this increase in the number of the electrocardiograms might be responsible for the augmentation of the action currents of the diaphragm.

A simple calculation however overthrows this objection. Let us assume that the frequency of the heart in the cat (the same reasoning with somewhat other numbers holds true also for the rabbit) is about 180 per minute[1]), then there will be present in the curve over a length of 0.5 a second, mostly $0.5 \times \frac{180}{60} = 1.5$ and at most 2 electrocardiograms. Supposing that after the elimination of the vagi the heart accelerates from 180 to f.i. 240 or even 360 beats per minute, an acceleration of 100%, which will only be seldom, if ever, present, then we can expect to find in our curves over 0.5 a second $0.5 \cdot \frac{360}{60} = 3$ electrocardiograms, i.e. an apparent augmentation of mostly 1 or at utmost 1.5 per 0.5 second. So this would give an apparent augmentation of the frequency of the action currents of the diaphragm of 2 or 3 per second. From this reasoning it is clear that even with these numbers, which we took as unfavourably as possible, this factor, which undoubtedly exists, cannot explain the augmentation present in our experiments.

We think it therefore permissible to conclude that for the greatest part, the augmentation of the frequency of the action currents of the diaphragm after elimination of both vagi is due *to the elemination of the centripetal impulses, which normally travel along the vagi to the central nervous system and obviously exert an inhibitory influence on the respiratory movements, at least in the cat and the rabbit.*

Since the researches of HERING and BREUER it is wellknown that centripetal vagal impulses have an important influence on the respiratory movements, especially on the inspiration. The fact shown by our experiments gives clear and, as far as we know, until now unknown evidence of this influence.

September 1923. *Physiological Laboratory of the University of Utrecht.*

[1]) This assumed number is on the high side; for a smaller number our reasoning becomes yet more conclusive.

Géologie. — "*Description de Raniniens nouveaux des terrains tertiaires de Borneo*", par V. VAN STRAELEN.

(Présenté par M. le Prof. G. A. F. MOLENGRAAFF à la séance du 24 novembre 1923).

Les Raniniens décrits ci-dessus ont été recueillis par M. J. A. LOHR [1]), au cours d'une exploration effectuée dans la vallée de la rivière Toehoep, affluent de rive gauche du fleuve Barito, au S.E. de l'île Borneo. Ils font actuellement partie des collections du Musée géologique de la "Technische Hoogeschool" à Delft. M. le Professeur G. A. F. MOLENGRAAFF, directeur de ce Musée, a bien voulu attirer mon attention sur ces matériaux et me les confier pour étude.

Famille: *Raninidae* DANA 1852.
1. — Genre: *Ranina* LAMARCK 1818.
Sous Genre: *Hela* MÜNSTER 1840.
Ranina (*Hela*) *Molengraaffi* nov. sp.
(Fig. 1*a* et *b*).

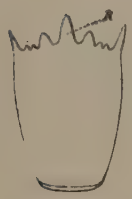


Fig 1*a*.

Fig. 1*b*.

Ranina (*Hela*) *Molengraaffi* nov. sp. — Grandeur naturelle.
1*a*. Face dorsale. — 1*b*. Face latérale droite. — R. Rostre.

Cette espèce est connue par les restes d'un seul individu, se présentant par la face tergale. Le céphalothorax dont la longueur dépasse la largeur d'environ $^1/_6$, s'élargit de l'arrière vers l'avant. Sa largeur mesurée au niveau de l'insertion des deux dents marginales et celle mesurée au bord postérieur, sont dans le rapport de 3 à 2. Le céphalothorax est bombé, la courbure s'accentuant dans la région médiane, au point de constituer une crête surbaissée. La région frontale s'incurve vers le haut, de sorte qu'elle semble précédée par

[1]) J. A. LOHR, *Mededeelingen over de Geologie der Doesoen-landen*. Verhandelingen van het Geologisch en Mijnbouwkundig Genootschap voor Nederland en Koloniën, Vergaderingen, N°. 45, 1914, pp. 174—175.

une faible dépression. Une autre dépression plus forte que la précédente, existe dans la région médiane du céphalothorax, au tiers postérieur. La région cardiaque est indiquée par une paire de sillons en arc de cercle, à concavité ouverte vers les bords latéraux.

Le bord frontal sensiblement rectiligne est occupé par un rostre triangulaire, large et long, bordé par des échancrures oculaires limitées chacune latéralement par un lobe triangulaire à base très large. Au delà de ces lobes, se trouvent deux petites épines et enfin une forte dent effilée et incurvée extérieurement, constituant le prolongement des bords latéraux. Ceux-ci sont un peu incurvés et à angle droit avec le bord postérieur. Ce dernier est à peu près rectiligne et bordé par un étroit sillon.

Le test paraissant lisse, est garni de fines granulations, légèrement acuminées, disposées sans ordre apparent.

La face sternale n'est pas connue.

L'attribution au genre *Ranina* pourrait être contestée, en se basant sur la petite taille, la simplicité du bord frontal et surtout le caractère de l'ornementation, fine au point que le test paraît lisse. A première vue, *R. Molengraaffi* se rapprocherait plutôt du genre *Notopus* De Haan, par la forme et l'ornementation du céphalothorax. Cependant, il lui manque entre autres caractères de *Notopus*, la crête épineuse située en arrière du bord frontal et unissant les deux dents latérales. Les autres genres de Raniniens à test lisse, dont ils constituent le groupe le plus nombreux, sont:

Raninoides H. Milne-Edwards Holocène,
Lyreidus De Haan, Oligocène-Holocène,
Notopoides Sp. Bate, Miocène et Holocène,
Cosmonotus Adams et White, Holocène,
Notosceles Bourne, Holocène,
Raninella A. Milne Edwards, Cénomanien-Sénonien,
Raninellopsis J. Boehm, Miocène,
Notopocorystes Mac' Coy, Cénomanien,
Eucorystes Bell, Albien-Cénomanien,
Palaeocorystes Bell, Albien-Cénomanien,
Hemioon Bell, Cénomanien,

et n'entrent pas en ligne de compte, à cause de la forme générale du céphalothorax et des caractères du bord frontal. Par le contour de son céphalothorax, *Notopus* est très voisin de *Ranina*.

M. R. Fabiani [1]) a distingué deux sous-genres dans *Ranina*, établis sur le caractère de l'ornementation. Le sous-genre *Lophoranina* réunit toutes les espèces dont le test est orné de côtes transversales épineuses et flexueuses, le sous-genre *Eteroranina* groupe les formes dont le test est soit à peu près lisse, soit orné de petits granules ou de petits tubercules acuminés, disposés en rangées et quelquefois sans ordre apparent. C'est pour des espèces appartenant à ce dernier groupe que G. zu Münster [2]) avait créé le genre *Hela*, dont le type *Hela speciosa* Münster provient du Chattien de Bünde. Je considère *Hela* comme synonyme de *Eteroranina* sur lequel il a la priorité.

Les *Ranina* décrites jusqu'à ce jour et qui se rapprochent le plus de celle trouvée à Bornéo, sont:

Ranina Ombonii Fabiani, de l'Yprésien des Colli Berici (Vicentin),

R. notopoides Bittner, du Lutétien du Monte Masua (Véronais),

R. budapestinensis Loerenthey, du Bartonien du Kis-Svábhegy (Hongrie),

R. Bouilleana A. Milne-Edwards, du Tongien de Biarritz (Aquitaine) et de Montecchio-Maggiore (Venétie),

R. granulosa A. Milne-Edwards, de l'Oligocène des environs de Dax (Aquitaine),

R. (*Hela*) *oblonga* Münster, du Chattien de Bünde (Hesse).

R. Molengraaffi se distingue:

de *R. Ombonii* par son céphalothorax moins long et plus large, beaucoup plus convexe, son bord frontal coïncidant à peu près avec la plus grande largeur de l'animal, enfin une ornementation beaucoup plus fine;

de *R. notopoides* par son bord frontal et son bord postérieur plus large, la présence d'une seule paire d'épines latérales, un rostre plus long et deux épines situées entre les lobes et l'épine latérale;

de *R. budapestinensis* par une forme beaucoup plus massive, le bord postérieur plus large, le rostre plus développé, les échancrures orbitaires plus profondes et les lobes correspondants plus développés, enfin des épines et des dents plus fortes;

de *R. oblonga* par son bord frontal plus étendu par rapport au bord postérieur et moins profondément découpé et une ornementation plus fine;

de *R. granulosa* par son bord frontal beaucoup moins découpé et le bord postérieur plus large.

[1]) R. Fabiani, *Sulle specie di Ranina finora note ed in particolare sulla Ranina Aldrovandii.* Atti dell' Academia scientifica Veneto-Trentino-Istriana, ser. 3a, t. 3, 1910, p. 85.

[2]) G. zu Munster, *Beiträge zur Petrefactenkunde*, Heft 3, 1840, p. 24.

Parmi toutes les espèces citées, c'est avec *R. budapestinensis* que *R. Molengraaffi* a le plus d'affinités.

Type. Musée géologique de la "Technische Hoogeschool" à Delft, échantillon n° 6 du lot K.A. 6491.

Gisement. Septaria argileux, légèrement calcarifère, coloré par de l'oxyde de fer, d'âge miocène d'après la carte de M. G. L. L. KEMMERLING. [1])

Localité. Vallée de la rivière Toehoep, entre l'embouchure de son affluent Bangkelan et Kampong Brawai (Borneo).

Je dédie cette espèce à M. G. A. F. MOLENGRAAFF, Professeur à la "Technische Hoogeschool" de Delft.

2. — Genre: *Raninella* A. MILNE EDWARDS 1862.
Raninella Toehoepae nov. sp.
(Fig. 2*a* et *b* et *c*).

Fig. 2*a*.

Fig. 2*b*.

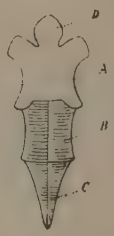


Fig. 2*c*.

Raninella Toehoepae nov. sp.

2*a*. Face dorsale, grandeur naturelle. — 2*b*. Face latérale droite, grandeur naturelle. — 2*c*. Plastron sternal, × 2. *A*, *B*, *C*. Sternites. *D*. Episternum. — *R*. Rostre.

Le céphalothorax est fortement bombé, s'élargissant considérablement vers l'avant, la plus grande largeur se trouvant à peu près à hauteur des sillons cardiaques et correspondant au double de la largeur du bord postérieur. Le bord frontal est faiblement convexe, porte un rostre droit à son origine et se terminant en une pointe triangulaire. De part et d'autre du rostre, le bord frontal présente des

[1]) G. L. L. KEMMERLING, *Geologisch-Topografische Schetskaart van het Stroomgebied der Barito (Borneo)*. Tijdschrift van het Koninklijk Nederlandsch Aardrijkskundig Genootschap, 2de ser., Deel 32 (1915), kaart N°. 6.

échancrures limitées par deux faibles épines, au delà desquelles se trouve une forte dent. Une dent marginale plus robuste encore, est insérée un peu au dessus de l'inflexion du bord latéral. Le bord postérieur est à peu près rectiligne, les bords latéraux sont convexes dans la région antérieure, mais rectilignes dans la région postérieure. Le bord postérieur et les bords latéraux postérieurs présentent un sillon marginal limitant une faible carène latérale.

La région cardiaque est marquée par deux sillons cardiaques, ayant à peu près la forme d'arcs de cercle à concavité ouverte vers les bords latéraux, surmontés chacun d'une paire de petits sillons parallèles.

Le plastron sternal est très large tout au moins dans ses parties antérieures. Le premier sternite placé entre la première paire de thoracopodes, est fort large et présente les deux entailles latérales et circulaires habituelles. Il se termine en avant par un épisternum arrondi. Le deuxième sternite est un peu moins large que le précédent, se rétrécissant vers l'arrière et pourvu d'un profond sillon médian. Le troisième sternite est étroit.

Le pléon se recourbe sous la face sternale. Sa largeur à l'origine est égale à celle du bord postérieur du céphalothorax. Ce qui reste des thoracopodes est trop fragmentaire pour permettre une description. L'ornementation du test est constituée par des granules extrêmement fins.

Le genre *Raninella* est un genre essentiellement crétacé. On en connait actuellement les espèces suivantes:

Raninella Trigeri A. Milne-Edwards, du Cénomanien du Mans (Sarthe),

R. elongata A. Milne-Edwards, du Cénomanien du Mans (Sarthe),

R. Schloenbachi Schlüter, du Sénonien (Emsien) de Wöltingerode (Saxe) [1]),

R. baltica Segerberg, du Danien de Faxe et d'Annetorp (Danemarck).

R. Toehoepae se distingue nettement des trois premières espèces citées, par la forme plus ovalaire de son bord frontal. Elle se rapproche de *R. baltica* dont le céphalothorax est également ovalaire, mais elle s'en distingue: 1° par son bord postérieur plus étroit, 2° son élargissement antérieur proportionellement plus considérable et reporté d'avantage vers l'arrière de l'animal.

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[1]) R. Schloenbachi est une espèce imparfaitement connue, basée sur un individu chez lequel la région frontale est en partie détruite et dont on ne connait que le moule interne des régions postérieures. Je la maintiens provisoirement dans le genre *Raninella*.

Jusqu'à présent le genre *Raninella* n'a été rencontré que dans le Crétacé moyen et supérieur. Il présente parmi les *Raninidae* un certain nombre de caractères que je considère comme primitifs: grande dimension du deuxième sternite et rétrécissement relativement faible des sternites postérieurs et du pléon. Il rappelle ainsi les genres *Palaeocorystes* Bell et *Notopocorystes* M'Coy du Gault du Kent, que je rattache aux *Raninidae* [1]).

Type. Musée géologique de la "Technische Hoogeschool" à Delft, échantillon K.A. 6504.

Cotypes. K.A. 6491, 6497, 6504, 6505, 6517, 6522.

Gisement. Septaria argileux, légèrement calcarifères, colorés par l'oxyde de fer, d'âge miocène d'après la carte de M. G. L. L. Kemmerling [2]).

Localités. Vallée de la rivière Toehoep, entre l'embouchure de son affluent Bangkelan et Kampong Brawai (Borneo).

Le nom spécifique est tiré de celui de la rivière Toehoep, affluent de gauche du Haut-Barito.

Les stratigraphes qui ont étudié les couches dans lesquelles la rivière Toehoep a creusé sa vallée, ne semblent pas d'accord sur leur âge. M. G. L. L. Kemmerling [3]) les rapporte au Miocène, M. J. A. Lohr [4]) hésite entre un âge anté — et post — éogène. Les deux Crustacés décapodes qui viennent d'être décrits ne permettent pas de trancher ce différend.

Qu'il soit cependant permis d'attirer l'attention sur le fait que *Ranina Molengraaffi*, forme lisse à bord frontal peu découpé et s'élargissant peu vers l'avant, à un cachet archaïque la rapprochant de ses congénères dont l'âge éogène et même crétacé n'est pas douteux. Quant à *Raninella Toehoepae*, elle appartient à un genre méso- et supracrétacé et présente d'ailleurs également des caractères primitifs accentués.

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[1]) V. van Straelen, *Note sur la position systématique de quelques Crustacés décapodes de l'époque crétacée.* Bulletins de l'Académie royale de Belgique, Classe des sciences, 1923, pp. 116—125, 6 fig.

[2]) G. L. L. Kemmerling, *Geologisch-Topografische schetskaart* etc., loc. cit.

[3]) G. L. L. Kemmerling, *Topografische en Geologische Beschrijving van het Stroomgebied van de Barito, in hoofdzaak wat de Doesoenlanden betreft.* Tijdschrift van het Koninklijk Nederlandsch Aardrijkskundig Genootschap, 2de ser., Deel 32, 1915, pp. 575—641 et pp. 717—774.

[4]) J. A. Lohr, loc. cit.

Mathematik. — *"Ueber die zu einem Punkte und einer Geraden gehörigen Polarkurven inbezug auf eine gegebene algebraische Kurve."* Von F. KÖLMEL in Baden-Baden.

(Mitgeteilt von Prof. JAN DE VRIES in der Sitzung vom 24 November 1923).

1. *Die Aufgabe.* Wird eine algebraische Kurve n-ter Ordnung durch eine Gerade in den n Punkten $R_1, R_2, \ldots R_n$ geschnitten, so ist nach JONQUIÈRES [1]) der harmonische Mittelpunkt r-ter Ordnung R zu diesen n Punkten und einem Zentrum O definiert durch die Gleichung

$$\binom{n}{r}\cdot\left(\frac{1}{OR}\right)^r-\binom{n-1}{r-1}\cdot\left(\frac{1}{OR}\right)^{r-1}\cdot\sum_1^n{}_i\left(\frac{1}{OR_i}\right)_1+\binom{n-2}{r-2}\cdot\left(\frac{1}{OR}\right)^{r-2}\cdot$$

$$\cdot\sum_1^n{}_i\left(\frac{1}{OR_i}\right)_2-\ldots+(-1)^n\cdot\binom{n-r}{0}\cdot\left(\frac{1}{OR}\right)^0\cdot\sum_1^n{}_i\left(\frac{1}{OR_i}\right)_n=0$$

wo $\binom{n}{k}$ Binomialkoeffizienten und $\sum_1^n\left(\frac{1}{OR_i}\right)_k$ die Summe der Produkte der reziproken Abschnitte OR_i zu je k bedeutet, $i=1, 2, \ldots r$.

Beschreibt die schneidende Gerade ein Strahlbüschel mit dem Zentrum Q, wahrend O eine Gerade p durchläuft, so beschreibt der harmonische Mittelpunkt r-ter Ordnung eine algebraische Kurve, die ich die zu dem Zentrum Q und der Geraden p gehörige Polarkurve r-ter Stufe inbezug auf die gegebene Grundkurve n-ter Ordnung nenne.

Allgemein lassen sich die Polarkurven auch auffassen als Erzeugnis des Strahlbüschels Q und des ihm projektiven Büschels der gewöhnlichen Polaren der Punkte der Geraden p.

2. Die vorliegende Mitteilung behandelt zunächst den Fall:

Die feste Grundkurve sei ein Kegelschnitt.

Hier kommt nur die Polarkurve erster Stufe in Betracht, da die zweiter Stufe identisch mit der gegebenen Kurve ist.

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[1]) Vgl. JONQUIÈRES. Mémoire sur la théorie des polaires etc. Journal de Liouville. 1857 oder
CREMONA. Geometrische Theorie der ebenen Kurven. Deutsche Ausgabe von Curtze, Greifswald 1865.

I. *Geometrisches.* Es sei f der gegebene Kegelschnitt, P der Pol von p inbezug auf f, q die Polare von Q und y die Polare des Schnittpunktes Y von p und q.

Um auf einem Strahle a von Q den gesuchten vierten harmonischen Punkt zu finden, schneiden wir a mit p (der Schnittpunkt sei $\mathfrak{A}$) und konstruieren zu $\mathfrak{A}$ die Polare a inbezug auf f, die durch P geht. Der Schnittpunkt A von $a \times a$ ist dann der gesuchte vierte harmonische Punkt. Daraus ergibt sich sofort: Jedem Strahl a des Strahlbüschels Q ist die konjugierte Polare a inbezug auf f durch den Punkt P zugeordnet, daher sind diese beiden Büschel projektiv und *der gesuchte Ort des vierten harmonischen Punktes ist ein Kegelschnitt. Diesen nenne ich den Polarkegelschnitt des Punktes Q und der Geraden p inbezug auf den gegebenen Kegelschnitt f und bezeichne ihn mit Φ.*

Aus dem obigen folgt, dass Q und p mit P bezw. q vertauschbar sind.

3. Die Φ geht jedenfalls durch die Schnittpunkte C, D von p mit f, ferner durch die Schnittpunkte U, V von q mit f, durch die Punkte Q und P und berührt die Geraden YQ und YP in Q bezw. P. XYZ ist das gemeinsame Polardreieck für f und Φ. Es ist auch leicht zu entscheiden, welcher Art der Kegelschnitt Φ ist. Soll nämlich Φ einen unendlich fernen Punkt B_∞ haben, so müssen die 2 entsprechenden Strahlen β und b der projektiven Büschel Q und P parallel sein, somit liegt der vierte harmonische Punkt in der Mitte der Schnittpunkte von β mit f.

Verbindet man diese Mitte mit M, so erhält man einen Strahl, der zu β konjugierte Polare ist. Konstruiert man also zu allen Strahlen β von Q die konjugierten Polaren durch M, so erhält man wieder ein zu dem Büschel Q projektives Büschel M und das Erzeugnis dieser zwei projektiven Büschel ist wieder ein $C_2 \equiv \lambda_1$, *der alle Sehnen in f, die durch Q gehen, halbiert.* Schneidet man λ_1 mit p, so geben die Verbindungsgeraden dieser Schnittpunkte mit Q die Richtungen der Asymptoten von Φ an. *Je nachdem also λ_1 die Gerade p in 2 Punkten schneidet, oder berührt oder gar nicht trifft, ist Φ eine Hyperbel, oder Parabel oder Ellipse.* λ_1 geht durch M, Q, U, V. Es gibt noch einen zweiten solchen entscheidenden Kegelschnitt λ_2, der durch M, P, C und D geht und analog wie λ_1 konstruiert wird. Dessen Schnitt mit q gibt dann die Entscheidung. λ_1 und λ_2 bleiben dieselben, wenn Q bezw. P erhalten bleibt, während p bezw. q sich ändert. Für alle möglichen Lagen von p bilden die λ_1 ein Netz von C_2 durch die Punkte Q, V, U; entsprechendes gilt für λ_2.

Auch der vierte Schnittpunt O von λ_1 mit Φ ist leicht anzugeben:

Man verbinde M mit P und schneide MP mit λ_1, der Schnittpunkt ist der gesuchte Punkt O. Denn PM und p sind konjugierte Richtungen inbezug auf f. Zieht man also $QO//p$ so sind QO und PO $(= MO)$ entsprechende Strahlen in den projektiven Büscheln Q und P bezw. Q und M; somit ist der Schnittpunkt von PM und QO sowohl ein Punkt von λ_1 als von Φ. Zieht man ferner QM und $PG//q$, so ist der Schnittpunkt dieser 2 Geraden sowohl ein Punkt von Φ als von λ_1. Entsprechendes gilt für λ_2.

Endlich kann man noch den Mittelpunkt M_3 von Φ bestimmen. Die Verbindungsgerade von Y mit der Mitte von QP geht durch M_3; ebenso die Verbindungsgerade der Mitten der Sehnen CD und QO und die Verbindungsgerade der Mitten von UV und PC. Auf den Durchmessern QM_3 und PM_3 lassen sich auch die Endpunkte E bezw. H bestimmen, die Tangenten in H und E sind dann parallel bezw. zu den Tangenten YQ und YP, so dass $YSTR$ ein dem Φ umschriebenes Parallelogramm ist.

4. Von besonderen Fällen je nach der Lage von Q und p seien kurz folgende erwähnt;

a. Ist p die unendlich ferne Gerade, so wird $P \equiv M$, $\Phi \equiv \lambda_1$.

b. Wenn p den f berührt, so berührt Φ den f in P und oskuliert λ_2 in diesem Punkte.

c. Wenn p durch M geht, hat Φ mindestens einen (reellen) unendlich fernen Punkt.

d. Wenn $p \equiv q$, so ist auch $P \equiv Q$ und Φ degeneriert in das von Q an f gehende Tangentenpaar. (Vgl. Analytisches.)

e. Wenn p durch Q geht, so liegt P auf q und Φ zerfällt in p bzw. q. (Gewöhnliche Polare.)

II. *Analytisches.*

Bezeichnungen.

5. Es seien:

$$f(x,y,z) \equiv a_{11}\,x^2 + 2\,a_{12}\,xy + a_{22}\,y^2 + 2\,a_{13}\,xz + 2\,a_{23}\,yz + a_{33}\,z^2 = 0 \quad (1)$$

die Gleichung des festen Kegelschnittes f, ebenso

$$g(x,y,z) \equiv b_{11}\,x^2 + 2\,b_{12}\,xy + b_{22}\,y^2 + 2\,b_{13}\,xz + 2\,b_{23}\,yz + b_{33}\,z^2 = 0 \quad (2)$$

die Gleichung eines zweiten Kegelschnittes g; $F(u, v, w)$ und $G(u, v, w)$ die adjungierten Formen zu f, bezw. g;

A und B die Determinanten von f, bezw. g; A_{ik}, B_{ik} die Unterdeterminanten von A und B,

$$3\,\theta = \sum_{i,k=1,2} a_{ik} B_{ik},\ 3\,H = \sum_{i,k=1,2} b_{ik} A_{ik} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

die beiden simultanen Invarianten von f und g,

$$H = \Sigma\left(\frac{\partial F}{\partial a_{ik}} \cdot b_{ik}\right) = \Sigma\left(\frac{\partial G}{\partial b_{ik}} \cdot a_{ik}\right) = H(u,v,w) = H_{11}u^2 + 2H_{12}uv + \left.\begin{array}{r} \\ \end{array}\right\} (4)$$
$$+ H_{22}v^2 + 2H_{13}uv + 2H_{23}vw + H_{33}w^2$$

die simultane Contravariante zu f und g, ferner x_0, y_0, z_0 die Koordinaten von Q, $\bar{x}_0$, $\bar{y}_0$, $\bar{z}_0$ die von P, u_0, v_0, w_0 die L. K. von q und $\bar{u}_0$, $\bar{v}_0$, $\bar{w}_0$ die L. K. von p, sodass

$$u_0 = f_1(x_0),\ v_0 = f_2(y_0),\ w_0 = f_3(z_0);\quad \bar{u}_0 = f_1(\bar{x}_0),\ \bar{v}_0 = f_2(\bar{y}_0),\ \bar{w}_0 = f_3(\bar{z}_0)$$

und umgekehrt:

$$x_0 = F_1(u_0),\ y_0 = F_2(v_0) \text{ u.s.w.}$$

Dabei ist

$$f_1(x) = \frac{\partial f(x,y,z)}{\partial x},\ f_2(y) = \frac{\partial f(x,y,z)}{\partial y},\ f_3(z) = \frac{\partial f(x,y,z)}{\partial z};$$

$$f_1(x_0) = \left.\frac{\partial f(x,y,z)}{\partial x}\right|_{x=x_0,\,y=y_0,\,z=z_0};\quad f_2(y_0) = \left.\frac{\partial f}{\partial y}\right|_{x=x_0,\,y=y_0,\,z=z_0}$$

$$F_1(u) = \frac{\partial F(u,v,w)}{\partial u};\quad F_1(u_0) = \left.\frac{\partial F(u,v,w)}{\partial u}\right|_{u=u_0,\,v=v_0,\,w=w_0},$$

woraus die übrigen Bezeichnungen sich von selbst ergeben.
Dann ist auch

$$\Sigma H_{ik}a_{ik} = 6\theta,\ \Sigma H_{ik}b_{ik} = 6H, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

und

$$\left.\begin{array}{l} B_{11}a_{11} + B_{12}a_{12} + B_{13}a_{13} + H_{11}b_{11} + H_{12}b_{12} + H_{13}b_{13} = 3\theta \\ A_{11}b_{11} + A_{12}b_{12} + A_{13}b_{13} + H_{11}a_{11} + H_{12}a_{12} + H_{13}a_{13} = 3H \end{array}\right\} . \quad (6)$$

6. Für einen Punkt R der Polarkurve erster Stufe zu O und p gilt dann

$$\frac{2}{OR} = \frac{1}{OR_1} + \frac{1}{OR_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

wo O der Schnittpunkt eines Strahles des Büschels Q mit p, R_1, R_2 die Schnittpunkte mit f sind. Sind ξ, η, ζ die Koordinaten von O, so folgt aus Obigem:

$$\left.\begin{array}{l} \dfrac{\lambda_1}{(x-\xi) + \lambda_1(x_0-\xi)} + \dfrac{\lambda_2}{(x-\xi) + \lambda_2(x_0-\xi)} = 0, \\ \dfrac{\lambda_1}{(y-\eta) + \lambda_1(y_0-\eta)} + \dfrac{\lambda_2}{(y-\eta) + \lambda_2(y_0-\eta)} = 0, \end{array}\right\} \quad . \quad . \quad (8)$$

wobei λ_1, λ_2 die Wurzeln der Gleichung

$$f(x_0,y_0,z_0) + \lambda \cdot \{f_1(x_0) \cdot x + f_2(y_0) \cdot y + f_3(z_0) \cdot z\} + \lambda^2 \cdot f(x,y,z) = 0 \quad (9)$$

sind, und für ξ,η,ζ die Gleichung besteht:

$$\overline{u_0}\,\xi + \overline{v_0}\,\eta + \overline{w_0}\,\xi = 0.$$

Durch Elimination von λ,ξ,η,ζ erhält man als Gleichung für die Polarkurve erster Stufe:

$$\left.\begin{aligned} &2f(x,y,z) \cdot (\overline{u_0}\,x_0 + \overline{v_0}\,y_0 + \overline{w_0}\,z_0) - \\ &-\{f_1(x_0) \cdot x + f_2(y_0) \cdot y + f_3(z_0) \cdot z\} \cdot \{\overline{u_0}\,x + \overline{v_0}\,y + \overline{w_0}\,z\} = 0 \equiv \Phi(x,y,z) \end{aligned}\right\} \quad (10)$$

Unter Anwendung der oben angegebenen Beziehungen zwischen den u_0,v_0,w_0 und x_0,y_0,z_0 kann man der Gleichung noch verschiedene andere Formen geben, von denen wir gelegentlich am passenden Ort Gebrauch machen werden. Erwähnt sei nur folgende Form:

$$\begin{aligned} \Phi(x,y,z) \equiv &\{f_1(x) \cdot x + f_2(y) \cdot y + f_3(z) \cdot z\} \cdot \{f_1(\overline{x_0})\,x_0 + f_2(\overline{y_0})\,y_0 + f_3(\overline{z_0}) \cdot z_0\} \\ &- \{f_1(x_0)\,x + f_2(y_0) \cdot y + f_3(z_0) \cdot z\} \cdot \{f_1(\overline{x_0})\,x + f_2(\overline{y_0})\,y + f_3(\overline{z_0}) \cdot z\} = 0. \end{aligned}$$

aus der die Vertauschbarkeit von Q und P besonders evident ist.

7. Zunächst ersieht man, dass die Φ-Kurve durch den Schnitt von f mit p geht, und dass sie, wenn Q auf p liegt, in p und q zerfällt; auch die Vertauschbarkeit von Q und p mit P und q ergiebt sich mit Rücksicht auf die in (5) gegebenen Beziehungen.

Ist

$$(x\,z_0 - x_0 z) + \lambda\,(y z_0 - y_0 z) = 0 \quad . \quad . \quad . \quad . \quad . \quad (11)$$

das Strahlbüschel Q, so ist das Strahlbüschel der Polaren zu den Schnittpunkten mit p gegeben durch

$$\left.\begin{aligned} &\{f_1(x) \cdot \overline{v_0}\,x_0 + f_2(y) \cdot (-\overline{u_0}\,x_0 + \overline{w_0}\,z_0) + f_3(z) \cdot \overline{v_0}\,z_0\} \\ &+ \lambda \cdot \{f_1(x) \cdot (\overline{v_0}\,y_0 + \overline{w_0}\,z_0) - f_2(y) \cdot \overline{u_0}\,y_0 - f_3(z) \cdot \overline{u_0}\,z_0\} = 0 \end{aligned}\right\} \quad (12)$$

Durch Elimination von λ erhält man wieder $\Phi(x,y,z) = 0$. Aus der Gleichung für Φ lassen sich die Gleichungen für die Kurven λ_1 und λ_2 in (2) ableiten. Diese sind nämlich spezielle Φ-Kurven, wenn p, bzw. q zur unendlich fernen Geraden wird. Nehmen wir Cartesische Koordinaten, sodass $z = 0$ die Gleichung der unendlich fernen Geraden ist, so haben wir:

$$\lambda_1(x,y,z) \equiv 2f(x,y,z) \cdot z_0 - \{f_1(x) \cdot x_0 + f_2(y) \cdot y_0 + f_3(z) \cdot z_0\} \cdot z = 0 \quad (13)$$

$$\lambda_2(x,y,z) \equiv 2f(x,y,z) \cdot z_0 - (\overline{u_0}\,x + \overline{v_0}\,y + \overline{w_0}\,z) \cdot z = 0.$$

Für den Schnittpunkt O von λ_1 mit Φ haben wir:

$$f_1(x_0) \cdot x + f_2(y_0) \cdot y + f_3(z_0)\, z = 0,$$

und

$$\overline{u_0}\, x + \overline{v_0}\, y - \overline{w_0}\, z \cdot (\overline{u_0}\, x_0 + \overline{v_0}\, y_0) = 0;$$

letzteres ist die durch Q gehende Parallele zu p. Für die Schnittpunkte der beiden Kurven λ_1 und λ_2 findet man:

a) $z = 0$,

b) $\overline{z_0}(u_0 x + v_0 y + w_0 z) - z_0 \cdot (\overline{u_0} x + \overline{v_0} y + \overline{w_0} z) = 0$,

d.h. b) geht durch den Mittelpunkt M und den Schnittpunkt V von p und q.

8. Die Φ-Kurve lässt sich auch auf folgende andere Arten erzeugen: Die Gleichung

$$f(x,y,z) + \lambda^2 \cdot (\overline{u_0}\, x + \overline{v_0}\, y + \overline{w_0}\, z)^2 = 0 \quad . \quad . \quad . \quad . \quad (14)$$

stellt ein Büschel von C_2 dar, die f in den Punkten C und D berühren. Das Büschel der Polaren des Punktes Q in Bezug auf dieses C_2-Büschel ist dann gegeben durch

$$\left.\begin{aligned} &f_1(x_0) \cdot x + f_2(y_0) \cdot y + f_3(z_0)\, z \\ &+ 2\lambda^2 \cdot (\overline{u_0}\, x + \overline{v_0}\, y + \overline{w_0}\, z)(\overline{u_0}\, x_0 + \overline{v_0}\, y_0 + \overline{w_0}\, z_0) = 0 \quad . \quad . \quad . \quad . \end{aligned}\right\} \quad (14a)$$

Durch Elimination von λ^2 ergiebt sich wieder Φ; ebenso aus dem Büschel

$$f(x,y,z) + \mu^2 \cdot (u_0 x + v_0 y + w_0 y)^2 = 0 \quad . \quad . \quad . \quad . \quad (15)$$

und dem zugehörigen Polarenbüschel für P in Bezug auf dieses Büschel.

Wenn man endlich die beiden Büschel in den Gleichungen (14) und (15) in Beziehung setzt durch die Relation:

$$2\lambda\mu(\overline{u_0}\, x_0 + \overline{v_0}\, y_0 + \overline{w_0}\, z_0) = 1, \quad . \quad . \quad . \quad . \quad . \quad (16)$$

so erhält man durch Elimination von λ, μ wiederum $\Phi(x, y, z) = 0$, neben einer zweiten, ebenso gebauten Gleichung.

9. Eine wichtige metrische Beziehung für die Punkte der Φ-Kurve ergiebt sich durch folgende Überlegung:

Es seien x, y, z die rechtwinkligen Koordinaten eines Punktes A, dann ist die Polare desselben in Bezug auf $f = 0$:

$$f_1(x) \cdot X + f_2(y) \cdot Y + f_3(z) \cdot Z = 0,$$

wenn X, Y, Z die laufenden Koordinaten sind. Somit ist der Abstand d_1 des Punktes A von seiner Polaren:

$$d_1 = \frac{f_1(x) \cdot x + f_2(y) \cdot y + f_3(z) \cdot z}{\sqrt{f_1^2(x) + f_2^2(y)}} = \frac{2f(x,y,z)}{\sqrt{f_1^2(x) + f_2^2(y)}}.$$

Der Abstand des Punktes A von p ist

$$d_2 = \frac{\overline{u}_0 x + \overline{v}_0 y + \overline{w}_0 z}{\sqrt{\overline{u}_0^2 + \overline{v}_0^2}}.$$

Der Abstand des Punktes Q von der obigen Polaren des Punktes A ist

$$n_1 = \frac{f_1(x) \cdot x_0 + f_2(y) \cdot y_0 + f_3(z) \cdot z_0}{\sqrt{f_1^2(x) + f_2^2(y)}}$$

und der Abstand des Punktes Q von p ist

$$n_2 = \frac{\overline{u}_0 x_0 + \overline{v}_0 y_0 + \overline{w}_0 z_0}{\sqrt{\overline{u}_0^2 + \overline{v}_0^2}}.$$

Setzt man nun $\frac{d_1}{d_2} = \frac{n_1}{n_2}$ und bringt nach Weghebung gemeinsamer Faktoren alles auf eine Seite, so erhält man wieder die Gleichung $\Phi(x, y, z) = 0$.

Somit ist $\Phi = 0$ der geometrische Ort des Punktes x, y, z, für den das Verhältnis der Abstände von seiner Polaren und von einer gegebenen Geraden p gleich ist dem Verhältnis der Abstände eines gegebenen Punktes Q von denselben zwei Geraden, absolut genommen.

10. Das dualistische Gegenbild der Φ-Kurve erhält man auf folgende Art: Die Strahlen des Büschels Q schneiden auf p eine Punktreihe aus, ebenso die des projektiven Büschels der konjugierten Polaren durch P auf q. Die Verbindungsgeraden der entsprechenden Punkte dieser zwei projektiven Punktreihen erzeugen einen Kegelschnitt Ψ; die gemeinsamen Tangenten von f und Ψ sind die Tangenten von f in C, D, U, V; Ψ hat mit f und Φ dasselbe Polardreieck gemeinsam.

11. *Büschel von Grundkurven und zugehörigen Polarkurven.*

Die 4 Punkte C, D, U, V bestimmen ein Büschel von C_3: $kg - f = 0$. Nimmt man für die Φ-Kurve eines jeden C_3 jeweils die in ein Geradenpaar zerfallenden C_2 des Büschels als p- und q-Gerade an, so gehören zu jedem C_3 des Büschels 3 Φ-Kurven, und umgekehrt.

Da diese ebenfalls durch C, D, U, V gehen, so muss $\Phi(x, y, z)$ von der Form $\mu g - f$ sein und es muss sich μ aus k und dem zu den zerfallenden Kurven gehörigen Parameter λ der Gleichung

$$C(\lambda) \equiv B\lambda^3 - 3\theta\lambda^2 + 3H\lambda - A = 0 \quad . \quad . \quad . \quad (17)$$

bestimmen lassen. Die Beziehung zwischen k, λ, μ erhält man auf folgende Weise. Es ist

$$\Phi(kg - f) = 2(kg - f) \cdot (\bar{u}_0 x_0 + \bar{v}_0 y_0 + \bar{w}_0 z_0) -$$
$$- (u_0 x + v_0 y + w_0 x) \cdot (\bar{u}_0 x + \bar{v}_0 y + \bar{w}_0 z) = 0$$

oder $= 2(kg - f) \cdot (\bar{u}_0 x_0 + \bar{v}_0 y_0 + \bar{w}_0 z_0) -$
$$- \{(kg_1 - f_1) \cdot x_0 + (kg_2 - f_2) \cdot y_0 + (kg_3 - f_3) \cdot z_0\} \cdot$$
$$\cdot (\bar{u}_0 x + \bar{v}_0 y + \bar{w}_0 z) = 0.$$

Zur Berechnung von $\bar{u}_0 x_0 + \bar{v}_0 y_0 + \bar{w}_0 z_0$ haben wir:

$$x_0 = k^2 \cdot G_1(u_0) - k \cdot H_1(u_0) - F_1(u_0)$$
$$= 2k^2 \cdot (B_{11} u_0 + B_{12} v_0 + B_{13} w_0) - 2k \cdot (H_{11} u_0 + H_{12} v_0 + H_{13} w_0)$$
$$+ 2(A_{11} u_0 + A_{12} v_0 + A_{13} w_0)$$

und zwei entsprechende Gleichungen für y_0 und z_0.

Zur Elimination von u_0, v_0, w_0 und $\bar{u}_0, \bar{v}_0, \bar{w}_0$ vergleicht man das Produkt $(u_0 x + v_0 y + w_0 z) \cdot (\bar{u}_0 x + \bar{v}_0 y + \bar{w}_0 z)$ mit $\lambda g - f$.

Dadurch findet man:

$$\left.\begin{aligned}(u_0 x + v_0 y + w_0 z) \cdot (\bar{u}_0 x + \bar{v}_0 y + \bar{w}_0 z) = \\ = 4 \cdot (Bk^3 - 3\theta k^2 + 3Hk - A) \cdot (\lambda g - f)\end{aligned}\right\} \quad (18)$$

und

$$u_0 x + \bar{v}_0 y_0 + \bar{w}_0 z_0 = 6H\lambda - 2A - 12\theta k\lambda + 12Hk + 2Bk^3\lambda - 6\theta k^2 \quad (19)$$

und endlich daraus dann:

$$\mu = \frac{A(-k+\lambda) - 3\theta k^2 \cdot (k+\lambda) + 6Hk^2}{Bk^2(-k+\lambda) + 3H(k+\lambda) - 6\theta k\lambda} \quad . \quad . \quad . \quad (20)$$

oder

$$\lambda = k \cdot \frac{(Bk^3\mu - A) - 3(\theta k^2 + H\mu) + 6Hk}{(Bk^3\mu - A) + 3(\theta k^2 + H\mu) - 6\theta k\mu} \quad . \quad . \quad . \quad (20a)$$

neben $C(\lambda) = 0$.

Setzt man hierin $k = \lambda$, so erhält man $\mu = \lambda$. D. h. nur dann ist $\Phi(\Phi) \equiv f$, wenn f und folglich auch Φ eine zerfallende C_3 des Büschels sind. Geometrisch erhellt dies, wenn man beachtet, dass die Pole P und Q sich auf den Seiten des dem Büschel gemein-

samen Polardreiecks XYZ bewegen. Die Schnittpunkte der Tangenten in U und V z. B. an f und Φ bestimmen auf der Seite XZ des Polardreiecks zwei coincidente projektive Punktreihen, deren Doppelpunkte eben die Schnittpunkte der zerfallenden Kurven des Büschels sind. Setzt man den für λ in $(20a)$ gegebenen Wert in die Gleichung $C(\lambda) = 3$ ein, so hat man eine Relation zwischen k und λ.

12. *Netz und Büschel von Polarkurven bei fester Grundkurve f.*

Hält man f und p fest, so bildet die Gesamtheit der Φ-Kurven ein Netz mit den Stützpunkten P, C, D. Jede solche C_2 ist aber nur Polarkurve für *einen* Punkt auf ihr, nämlich den Pol für die gemeinsame zweite Sehne von f und Φ. Macht man die Tangenten von f in C und D bezw. zur X- und Y-Achse und die Berührungssehne CD zur Z-Achse, so wird

$$f(x, y, z) \equiv xy + z^2 = 0$$

und

$$\Phi(x, y, z) = 2 z_0 xy - y_0 xz - x_0 yz = 0.$$

Die Gleichung einer C_2 durch P, C, D hat dann die Form $\alpha xy + \beta xz + \gamma yz = 0$; daraus folgt für den Pol $x_0 : y_0 : z_0 = \alpha : -2\beta : -2\gamma$. Beschreibt nun der Pol eine Gerade

$$Q(x_0, y_0, z_0) \equiv ux_0 + vy_0 + wz_0 = 0,$$

wo x_0, y_0, z_0 die laufenden Koordinaten sind, so kann man das Büschel der zugehörigen Φ-Kurven in der Form schreiben:

$$x_0 (u x z - v y z) + z_0 . (2 v x y + w x z) = 0.$$

Für den vierten Grundpunkt dieses Büschels hat man also:

$$u x - v y = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

und

$$2 v y + w z = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21a)$$

woraus folgt

$$u x + v y + w z = 0.$$

(21) ist die lineare Polare des Schnittpunktes von p mit der Geraden $Q(x, y, z) = 0$. Somit liegt der vierte Schnittpunkt auf der Geraden $Q(x, y, z) = 0$ und eben dieser Polaren.

13. Beschreibt der Pol Q einen Kegelschnitt:

$$Q(x_0, y_0, z_0) \equiv c_{11} x^2 + 2c_{12} xy + c_{22} y^2 + 2 c_{13} xz + 2 c_{23} yz + c_{33} z^2 = 0, \quad (22)$$

während f, P und ρ festbleiben, so erhält man für die Enveloppe des Büschels der Φ-Kurven die Gleichung:

$$\left.\begin{aligned} E(\xi,\eta,\zeta) \equiv 4\,C_{33}\xi^2\eta^2 - 4\,C_{23}\xi^2\eta\zeta - 4\,C_{13}\xi\eta^2\zeta + C_{22}\xi^2\zeta^2 + \\ + 2\,C_{12}\xi\eta\zeta^2 + C_{11}\eta^2\zeta^2 = 0 \end{aligned}\right\} \quad (23)$$

wo die C_{ik} die Unterdeterminanten zu den c_{ik} sind. Die E ist also eine rationale Kurve vierter Ordnung mit den 3 Doppelpunkten [1]) in den Punkten P, C, D.

Die Q und E berühren sich in den 4 Punkten, die gegeben sind durch die Gleichungen:

$$\left.\begin{aligned} c_{11}x + c_{12}y + c_{13}z &= \varrho \cdot yz \\ c_{12}x + c_{22}y + c_{23}z &= \varrho \cdot xz \\ c_{13}x + c_{23}y + c_{33}z &= -\varrho \cdot 2xy \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Durch $\Phi \equiv 2\xi\eta z_0 - \xi\zeta y_0 - \eta\zeta x_0 = 0$ ist jedem Punkte x_0, y_0, z_0 auf Q ein Punkt ξ, η, ζ auf E zugeordnet und umgekehrt. Der Übergang von x_0, y_0, z_0 auf Q geschieht, indem man, wie oben angegeben, zu E übergeht und den Berührungspunkt von Φ und E bestimmt. Der Übergang von einem Punkte ξ, η, ζ auf E geschieht, indem man diesen als Pol betrachtet und durch das entsprechende Verfahren die Enveloppe der zugehörigen Φ-Kurven bestimmt, wenn ξ, η, ζ auf E wandert. Diese ist dann eben wieder die Q-Kurve und die doppeltgezählten Seiten des Dreiecks PCD (abgesehen von dem auftretenden Faktor Δ der Determinante der c_{ik}).

Zu einer anderen Darstellung dieser Berührungstransformation, die deren Bedeutung erst kennzeichnet, gelangt man durch folgende Überlegung: Es sei $E(\xi, \eta, \zeta) = 0$ gegeben, dann ist die Gleichung der Tangente in einem Punkte ξ, η, ζ:

$$E_1(\xi) \cdot x + E_2(\eta)\ y + E_3(\zeta) \cdot z = 0.$$

Soll nun eine C_3 transformiert werden, die E in diesem Punkte berührt und durch die Punkte C, D, P geht, so ist diese C_3 von der Form:

$$\mathfrak{A}_{12}xy + \mathfrak{A}_{13}xz + \mathfrak{A}_{23}yz = 0,$$

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[1]) Die Schnittpunkte der Tangenten in dem Doppelpunkt $x = y = 0$ liegen auf der C_2:

$$L(x, y, z) = (c_{13}x + c_{23}y + c_{33}z)^2 + 4\,c_{33}\,xy = 0.$$

Diese berührt die Tangenten von f in C und D in deren Schnittpunkten mit der gewöhnlichen Polaren des Punktes $x = 0,\ y = 0$ in Bezug auf Q.

daher muss sein:

$$\left.\begin{aligned} \mathfrak{A}_{11} &= \varrho \cdot \frac{E_3 \cdot \zeta^2 - E_1 \cdot \xi\zeta - E_2 \eta\zeta}{2\xi\eta\zeta} \\ {}_{12} &= \varrho \cdot \frac{E_2 \cdot \eta^2 - E_3 \eta\zeta - E_1 \cdot \xi\eta}{2\xi\eta\zeta} \\ \mathfrak{A}_{13} &= \varrho \cdot \frac{E_1 \cdot \zeta^2 - E_2 \xi\eta - E_3 \cdot \xi\zeta}{2\xi\eta\zeta} \end{aligned}\right\} \quad (25)$$

Indem man für E_1, E_2, E_3 die Werte einsetzt, erhält man als Abbildungskurve:

$$\left.\begin{aligned} &2 \cdot (-2 C_{23} \xi\eta + C_{22} \xi\zeta + C_{12} \eta\zeta) \cdot xy \\ &-(-2 C_{23} \xi\eta + C_{22} \xi\zeta + C_{13} \eta\zeta) \cdot xz \\ &-(-2 C_{13} \xi\eta + C_{12} \xi\zeta + C_{11} \eta\zeta) \cdot yz = 0, \end{aligned}\right\} \quad (26)$$

oder abgekürzt:

$$2 U_3 xy - U_2 xz - U_1 yz = 0.$$

Durch Vergleich mit der früheren Form der Φ-Kurve ergiebt sich die Beziehung:

$$x_0 : y_0 : z_0 = U_1 : U_2 : U_3, \quad (27)$$

woraus durch Auflösung nach ξ, η, ζ folgt:

$$\xi : \eta : \zeta = V_3 V_2 : V_3 V_1 : V_1 V_2, \quad (27a)$$

wobei

$$V_1 = 2 Q_1 (x_0), \quad V_2 = 2 Q_2 (y_0), \quad V_3 = Q_3 (z_0).$$

Somit vermittelt unsere Φ-Kurve eine birationale quadratische Transformation. Daraus erhellt jetzt auch, dass E rational sein muss. Die Gleichung $\Phi (x_0, y_0, z_0; \xi, \eta, \zeta) = 0$ und die Gleichungen (27) und (27a) zwischen den x_0, y_0, z_0 und den ξ, η, ζ sind also äquivalent.

14. Es soll hier noch gezeigt werden, dass diese Abbildung eine spezielle Berührungstransformation repräsentiert, ohne auf das zuletzt Auseindergesetzte Bezug zu nehmen. Es seien zwei E-Kurven gegeben

$$\left.\begin{aligned} E^{(1)} \equiv 4 C_{33} x^2 y^2 - 4 C_{23} x^2 yz - 4 C_{13} x y^2 z + C_{22} x^2 z^2 + \\ + 2 C_{12} x y z^2 + C_{11} y^2 z^2 = 0, \end{aligned}\right\} \quad (28)$$

$$E^{(2)} \equiv 4 D_{33} x^2 y^2 - 4 D_{23} x^2 y z - \ldots + D_{11} y^2 z^2 = 0. \quad (28a)$$

Soll dann einem Punkte ξ, η, ζ, der beiden E-Kurven gemeinsam ist, derselbe Punkt x_0, y_0, z_0 entsprechen, so muss sein:

$$\left.\begin{aligned} -2C_{22}\xi\eta + C_{23}\xi\zeta + C_{12}\eta\zeta &= \varrho\ (-2D_{22}\zeta\eta + D_{23}\xi\zeta + D_{12}\eta\zeta) \\ -2C_{23}\xi\eta + C_{33}\xi\zeta + C_{13}\eta\zeta &= \varrho\,.(-2D_{23}\xi\eta + D_{33}\xi\zeta + D_{13}\eta\zeta) \\ -2C_{12}\xi\eta + C_{13}\xi\zeta + C_{11}\eta\zeta &= \varrho\,.(-D_{12}\xi\eta + D_{13}\xi\zeta + D_{11}\eta\zeta) \end{aligned}\right\}. \quad (29)$$

Nun ist:

$$\left.\begin{aligned} 2E^{(1)}(\xi,\eta,\zeta) - E_1^{(1)}(\xi)\,.\,\xi &= -2\xi\zeta\,.(-2C_{12}\xi\eta + C_{13}\xi\zeta + C_{11}\eta\zeta) \\ 2E^{(2)}(\xi,\eta,\zeta) - E_1^{(2)}(\xi)\,.\,\xi &= -2\xi\zeta\,.(-2D_{12}\xi\eta + D_{13}\xi\zeta + D_{11}\eta\zeta) \end{aligned}\right\} \quad (30)$$

und entsprechende Gleichungen bestehen für die Ableitungen nach η und ζ.

Wegen $E^{(1)}(\xi, \eta, \zeta) = E^{(2)}(\xi, \eta, \zeta) = 0$, ergiebt sich also: $E_1^{(1)} = E_1^{(2)}$; $E_2^{(1)} = E_2^{(2)}$; $E_3^{(1)} = E_3^{(2)}$. Dies sind aber gerade die Bedingungen für die Berührung von $E^{(1)}$ und $E^{(2)}$ im Punkte ξ, η, ζ. Ebenso ergiebt sich, wenn man statt der ξ, η, ζ die x_0, y_0, z_0 einführt, dass die den $E^{(1)}$ un $E^{(2)}$ entsprechenden Kurven $Q^{(1)}$ und $Q^{(2)}$ sich in dem Punkte x_0, y_0, z_0 berühren.

Jede C_2, die f in C und D berührt, geht in sich über, indem die entsprechende E-Kurve in diese und die zwei Tangenten an f in den Punkten C und D zerfällt. Wenn die D-Kurve Q im Punkte x_0, y_0, z_0 berührt, so geht auch die E-Kurve durch diesen Punkt und berührt die Q-Kurve daselbst.

Mathematics. — *„Ueber den natürlichen Dimensionsbegriff."* [1]) By Prof. L. E. J. BROUWER.

(Communicated at the meeting of November 24, 1923).

Auf Grund der Invarianz der Dimensionenzahl [2]) lässt sich die Dimensionenzahl einer Mannigfaltigkeit [3]) definieren als die Anzahl der Parameter, durch welche sich die Mannigfaltigkeit in der Umgebung eines beliebigen ihrer Punkte eineindeutig und stetig darstellen lässt. Diese „arithmetische" Definition trägt aber nach POINCARÉ [4]) unserer intuitiven Raumanschauung ungenügend Rechnung. POINCARÉ erhebt deshalb die Forderung einer rekurrenten Definition von etwa folgender Form [5]):

„Ein Kontinuum heisse n-dimensional, wenn man es durch ein oder mehrere (n—1)-dimensionale Kontinua in getrennte Stücke zerlegen kann."

Obgleich der *n*-dimensionale JORDANsche Satz [6]) auf die Möglichkeit einer derartigen Definition deutet, so lässt sich diese in der zitierten Form dennoch nicht aufrecht erhalten.

Zunächst bemerken wir, dass das Wort *„Kontinuum"* hier sicher nicht etwa im Sinne von *„Mannigfaltigkeit"* aufgefasst werden darf; in diesem Falle würde nämlich die Definition erst brauchbar werden, nachdem eine von der Parameterdarstellung unabhängige Charakterisierung der Mannigfaltigkeiten unter den abstrakten Mengen gelungen sein würde. Weil dies aber bis jetzt nicht der Fall ist, so müsste der POINCARÉschen Definition irgendeine allgemeinere abstrakte Charakterisierung des Kontinuums vorausgeschickt werden, z. B. diese: *„Eine Normalmenge* (im FRÉCHETschen Sinne) *π heisse ein Kontinuum, wenn es für je zwei ihrer Elemente m_1 und m_2 eine zusammenhän-*

[1]) Die vorliegende Mitteilung bildet bis auf den Inhalt von Fussnote [19]) und die in Fussnote [11]) angegebene Berichtigung einen Wiederabdruck meiner in 1913 im Journal für die reine und angewandte Mathematik (Bd. 142, S. 146—152) unter demselben Titel erschienenen Abhandlung.

[2]) Vgl. meinen Beweis in Math. Annalen 70, S. 161—165 und die daran anknüpfenden Entwicklungen von LEBESGUE in C. R. de l'Acad. des sciences, Paris, 27 mars 1911.

[3]) Für die Definition des Begriffes „Mannigfaltigkeit" vgl. Math. Annalen 71, S. 97.

[4]) Revue de métaphysique et de morale, 1912, S. 486, 487.

[5]) a. a. O., S. 488.

[6]) Vgl. den teilweise von LEBESGUE, teilweise von mir erbrachten Beweis in C. R. de l'Acad. des sciences. Paris, 27 mars 1911, und Math. Annalen 71, S. 305—319.

gende, abgeschlossene[7]) *Menge gibt, welche Teilmenge von* π *ist und* m_1 *und* m_2 *enthält.*[8]). Für solche allgemeinere Kontinua, welche keine Mannigfaltigkeiten sind, würde aber unsere Definition zu Schwierigkeiten führen; z. B. würde man einem Kegel des Cartesischen Raumes, der sich ja durch einen Punkt zerlegen lässt, nur *eine* Dimension zusprechen dürfen.

Auch die Worte *„ein oder mehrere"* könnten nicht unverändert beibehalten werden, weil mehrere m-dimensionale Mannigfaltigkeiten zusammen eine $(m + p)$-dimensionale Mannigfaltigkeit bilden können.

Alle diese Mängel lassen sich nun beseitigen, indem wir zunächst die Poincarésche rekurrente Definition wie folgt abändern:

Es sei π irgendeine Normalmenge[9]), π_1, ϱ und ϱ' drei Teilmengen von π, welche innerhalb π abgeschlossen[10]) sind und keine gemeinsamen Punkte besitzen. Alsdann heissen ϱ und ϱ' *in* π *durch* π_1 *getrennt*, wenn π_1 in π eine ϱ enthaltende, aber ϱ' nicht enthaltende Gebietsmenge g bestimmt.[11]) Der Ausdruck: *„*π *besitzt den allgemeinen Dimensionsgrad* n*"*, in welchem n eine beliebige natürliche Zahl bezeichnet, soll nun besagen, dass für jede Wahl von ϱ und ϱ' eine trennende Menge π_1 existiert, welche den allgemeinen Dimensionsgrad $n-1$ besitzt, dass aber nicht für jede Wahl von ϱ und ϱ' eine trennende Menge π_1 existiert, welche einen geringeren allgemeinen Dimensionsgrad als $n-1$ besitzt. Weiter soll der Ausdruck: *„*π *besitzt den allgemeinen Dimensionsgrad Null bzw. einen unendlichen*

[7]) Unter einer *abgeschlossenen* Menge verstehen wir hier eine ihre Grenzelemente enthaltende Menge, in welcher jede unendliche Folge von Elementen mindestens ein Grenzelement aufweist.

[8]) Diese Definition ist der von Schoenflies für die Kontinua des n-dimensionalen Raumes gegebenen nachgebildet (vgl. Bericht über die Lehre von den Punktmannigfaltigkeiten, Bd. II, S. 117).

[9]) Inwieweit die Definition des Textes auch für Mengen allgemeinerer Art einen naturgemässen Sinn behält, soll hier unerörtert bleiben.

[10]) Dieser Ausdruck besagt, dass π_1, ϱ und ϱ' alle ihre in π gelegenen Grenzpunkte enthalten.

[11]) Diesen der Gebietsmenge g auferlegten Bedingungen können natürlich mehrere Gebietsmengen von π genügen. Im in [1]) zitierten Original hat sich an dieser Stelle eine andere, mit dem übrigen Inhalte des Aufsatzes in keinem Zusammenhang stehende Trennungsdefinition eingeschlichen. Dass die obige (übliche) Definition die in der vorliegenden Abhandlung in Wirklichkeit gebrauchte ist, geht aus dem Zusammenhang hervor, insbesondere aus Fussnote[16]) und dem zugehörigen Passus des Textes. Die daselbst eingeführte, von π_2 in τ_1 bestimmte, an die Kante $E_1 E_2$ grenzende Gebietsmenge kann nämlich keinen anderen Sinn haben, als den des Durchschnittes einer schon vorhandenen von π_2 in π_1 bestimmten, an $E_1 E_2$ grenzenden, an $E_1 E_3 \ldots E_{n+1}$ jedoch nicht grenzenden Gebietsmenge mit τ_1. Auf die Berichtigung, welche hier anzubringen war, bin ich von Herrn P. Urysohn in Moskau aufmerksam gemacht worden.

allgemeinen Dimensionsgrad" bedeuten, dass π kein Kontinuum als Teil enthält, bzw. dass zu π weder die Null noch irgendeine natürliche Zahl als ihr allgemeiner Dimensionsgrad gefunden werden kann.[12])

Dieser Definition lässt sich leicht eine von der Rekurrenz unabhängige Form geben. Dazu denken wir uns die Menge π von zwei Personen A und B der „*Dimensionsoperation*" unterzogen, worunter wir folgendes verstehen: A wählt in π zwei innerhalb π abgeschlossene Teilmengen ϱ und ϱ' beliebig aus, worauf B ϱ und ϱ' in π trennt durch eine innerhalb π abgeschlossene Menge π_1. Sodann wählt A in π_1 zwei innerhalb π_1 abgeschlossene Teilmengen ϱ_1 und ϱ'_1 beliebig aus, worauf B ϱ_1 und ϱ'_1 in π_1 trennt durch eine innerhalb π_1 abgeschlossene Menge π_2. Dieser Prozess wird unbeschränkt wiederholt, bis eventuell eine Menge π_h auftritt, welche kein Kontinuum mehr als Teil enthält. Wenn einerseits B unabhängig von den Wahlen der ϱ_ν und ϱ'_ν dafür sorgen kann, dass eine Menge π_h auftritt, deren $h \leqq n$, und andererseits A unabhängig von den Wahlen der π_ν dafür sorgen kann, dass *keine* Menge π_h auftritt, deren $h < n$, so werden wir sagen, dass π *den allgemeinen Dimensionsgrad* n *besitzt*. Wen dagegen keine natürliche Zahl n existiert mit der Eigenschaft, dass B unabhängig von den Wahlen der ϱ_ν und ϱ'_ν dafür sorgen kann, dass eine Menge π_h auftritt, deren $h \leqq n$, so werden wir sagen, dass π *einen unendlichen allgemeinen Dimensionsgrad besitzt.*

Wenn zu einem Punkte P von π Umgebungen, welche den allgemeinen Dimensionsgrad m, aber keine Umgebungen, welche einen geringeren allgemeinen Dimensionsgrad besitzen, existieren, so werden wir sagen, dass π *in* P *den Dimensionsgrad* m *besitzt.* In verschiedenen Punkten kann eine Menge verschiedene Dimensionsgrade besitzen: keiner von diesen kann indes den allgemeinen Dimensionsgrad der Menge übersteigen. Falls in jedem Punkte der Menge der Dimensionsgrad dem allgemeinen Dimensionsgrade der Menge gleich ist, so werden wir sagen, dass die Menge *einen homogenen Dimensionsgrad besitzt.*

Auf Grund der vorstehenden Definitionen soll nun die Poincaré-sche Forderung vollständig erfüllt werden durch die Begründung von folgendem

Dimensionssatz. *Eine n-dimensionale Mannigfaltigkeit besitzt den homogenen Dimensionsgrad* n.[13])

Zum Beweise dieses Satzes zeigen wir zunächst, dass B bei der

[12]) Nach dieser Definition wird sowohl für den Hilbertschen wie für den Fréchetschen R_n ein unendlicher allgemeiner Dimensionsgrad gefunden.

[13]) Weil der Dimensionsgrad offenbar eine Invariante der Analysis Situs ist, so ist im Dimensionssatz die Invarianz der Dimensionenzahl enthalten.

Dimensionsoperation dafür sorgen kann, dass $h \leqq n$. Dazu konstruiert B, nachdem A die Mengen ϱ und ϱ' bestimmt hat, eine simpliziale Zerlegung [14]) ξ von π, und zwar in solcher Weise, dass, wenn wir unter einem ${}_{\pi}s_{\xi}$ bzw. ${}_{\pi}s_{\xi'}$ ein entweder in seinem Inneren oder auf seiner Grenze Punkte von ϱ bzw. ϱ' enthaltendes Grundsimplex von ξ verstehen, kein ${}_{\pi}s_{\xi}$ mit einem ${}_{\pi}s_{\xi'}$ identisch ist und kein ${}_{\pi}s_{\xi}$ an ein ${}_{\pi}s_{\xi'}$ grenzt. Alsdann bilden diejenigen $(n-1)$-dimensionalen Seiten der ${}_{\pi}s_{\xi}$, welche weder in ihrem Inneren noch auf ihrer Grenze Punkte von ϱ enthalten, ein System von zweiseitigen $(n-1)$-dimensionalen Pseudomannigfaltigkeiten [15]), in welchem übrigens mehrere Elemente oder Elementseiten zusammenfallen können. Die von diesen Pseudomannigfaltigkeiten gebildete Punktmenge wählt B als π_1. Falls darauf A die Mengen ϱ_1 und ϱ'_1 in demselben Teilkontinuum von π_1 wählt, so konstruiert B eine solche simpliziale Zerlegung von π_1, dass kein ${}_{\pi_1}s_{\varrho_1}$ mit einem ${}_{\pi_1}s_{\varrho'_1}$ identisch ist und kein ${}_{\pi_1}s_{\varrho_1}$ an ein ${}_{\pi_1}s_{\varrho'_1}$ grenzt. Alsdann bilden diejenigen $(n-2)$-dimensionalen Seiten der ${}_{\pi_1}s_{\varrho_1}$, welche weder in ihrem Inneren noch auf ihrer Grenze Punkte von ϱ_1 enthalten, ein System von zweiseitigen $(n-2)$-dimensionalen Pseudomannigfaltigkeiten, in welchem übrigens wieder mehrere Elemente oder Elementseiten zusammenfallen können. Die von diesen Pseudomannigfaltigkeiten gebildete Punktmenge wählt B als π_2. In dieser Weise fortfahrend, gelangt B schliesslich zu einer Menge π_n, welche kein Kontinuum mehr als Teil enthält, es sei denn, dass der Prozess schon früher dadurch beendet wurde, dass A einmal ϱ_ν und ϱ'_ν nicht in demselben Teilkontinuum von π_ν wählte.

Wir zeigen zweitens, dass A bei der Dimensionsoperation dafür sorgen kann, dass h nicht kleiner als n ausfällt. Dazu wählt A in π von einem n-dimensionalen Elemente $E_1 E_2 \ldots E_{n+1}$ den Punkt E_1 als ϱ und die $(n-1)$-dimensionale Seite $E_2 \ldots E_{n+1}$ als ϱ'; den zur Elementseite $E_1 E_2$ bzw. $E_1 E_3 \ldots E_{n+1}$ gehörigen Teil von π_1 als ϱ_1 bzw. ϱ'_1; den zur Elementseite $E_1 E_2 E_3$ bzw. $E_1 E_3 E_4 \ldots E_{n+1}$ gehörigen Teil von π_2 als ϱ_2 bzw. ϱ'_2; usw. Um zu beweisen, dass von den Punktmengen $\pi_1, \pi_2, \ldots \pi_n$ keine in Fortfall kommen kann, bezeichnen wir mit τ das Ausgangselement $E_1 E_2 \ldots E_{n+1}$, mit τ_1 die Grenze des von π_1 in τ bestimmten, an den Punkt E_1 grenzenden Gebiets g, mit τ_2 die Grenze der von π_2 in τ_1 bestimmten, an die Kante $E_1 E_2$ grenzenden Gebietsmenge [16]) g_1, mit τ_3 die Grenze der

[14]) Math. Annalen 71, S. 101.

[15]) a. a. O., S. 305.

[16]) Unter einer in τ_ν gelegenen Gebietsmenge verstehen wir eine in τ_ν gelegene Punktmenge, von der kein Punkt Grenzpunkt der durch sie in τ_ν bestimmten Komplementärmenge ist.

von π_3 in τ_2 bestimmten, an die zweidimensionale Seite $E_1 E_2 E_3$ grenzenden Gebietsmenge g_3, usw., konstruieren in τ eine simpliziale Zerlegung von der Dichte ε[17]), bezeichnen mit γ das n-dimensionale Fragment[18]), welches von den mitsamt ihrer Grenze zu g gehörigen Grundsimplexen gebildet wird, mit σ_1 den *innerhalb* τ gelegenen Teil der gleichfalls simplizial zerlegt vorliegenden Grenze von γ, mit ε_1 das Maximum der Abstände, welche die Punkte von σ_1 von τ_1 besitzen, mit γ_1 das $(n-1)$-dimensionale Fragment, welches von denjenigen Grundsimplexen von σ_1, die von g_1 einen Abstand $\leqq \varepsilon_1$ besitzen, gebildet wird, mit σ_2 den *innerhalb* σ_1 gelegenen Teil der Grenze von γ_1, mit ε_2 das Maximum der Abstände, welche die Punkte von σ_2 von τ_2 besitzen, und fahren so fort. Alsdann konvergieren $\varepsilon_1, \varepsilon_2, \ldots \varepsilon_n$ mit ε gegen Null, so dass die eventuelle Existenz von $\sigma_1, \sigma_2, \ldots \sigma_n$ diejenige von $\tau_1, \tau_2, \ldots \tau_n$, mithin auch diejenige von $\pi_1, \pi_2, \ldots \pi_n$, in denen ja der Reihe nach $\tau_1, \tau_2, \ldots \tau_n$ als Teilmengen enthalten sind, nach sich ziehen wird.

Hiermit ist der Dimensionssatz zurückgeführt auf folgenden

Hilfssatz. Es sei σ ein simplizial zerlegtes n-dimensionales Element mit den Eckpunkten $E_1, E_2, \ldots E_{n+1}$; γ ein aus Grundsimplexen von σ gebildetes Fragment, das alle an E_1, aber kein an $E_2 E_3 \ldots E_{n+1}$ grenzendes Grundsimplex von σ enthält; σ_1 der innerhalb σ liegende Teil der Grenze von γ; γ_1 ein aus Grundsimplexen von σ_1 gebildetes Fragment, das alle an $E_1 E_2$, aber kein an $E_1 E_3 \ldots E_{n+1}$ grenzendes Grundsimplex von σ_1 enthält; σ_2 der innerhalb σ_1 liegende Teil der Grenze von γ_1; γ_2 ein aus Grundsimplexen von σ_2 gebildetes Fragment, das alle an $E_1 E_2 E_3$, aber kein an $E_1 E_2 E_4 \ldots E_{n+1}$ grenzendes Grundsimplex von σ_2 enthält; σ_3 der innerhalb σ_2 liegende Teil der Grenze von γ_2; usw. Alsdann kann von den Punktmengen $\sigma_1, \sigma_2, \sigma_3, \ldots \sigma_n$ keine verschwinden.

Dieser Hilfssatz, auf den schon Lebesgue in Math. Annalen 70 die Invarianz der Dimensionenzahl zurückgeführt hat, dessen Beweis daselbst aber eine wesentliche Lücke aufweist[19]), leuchtet unmittel-

[17]) Math. Annalen 71, S. 101.

[18]) a. a. O., S. 306.

[19]) Die „faits bien évidents", welche dieser Beweis (auf S. 167) voraussetzt, sind nämlich unrichtig, und bilden, wenn sie in eine richtige Form gebracht werden, eine Eigenschaft, welche tiefer liegt, als der Hilfssatz selbst. Nachdem Herr Lebesgue (in 1911) auf dieses Versehen hingewiesen worden war, teilte er mir seine Absicht mit, binnen kurzem im Bull. de la Soc. Math. de France einen neuen Beweis des Hilfssatzes zu bringen, von dem er mir gleichzeitig die Hauptzüge auseinandersetzte. Obgleich diese Auseinandersetzungen mich nicht befriedigten, meinte ich dennoch im in [1]) zitierten Original auf die von Herrn Lebesgue zugesagte Veröffentlichung hinweisen zu müssen. Dieselbe ist indes ausgeblieben und erst in Funda-

bar ein, wenn wir den von mir in Math. Annalen 71 [19]) eingeführten Begriff des *Abbildungsgrades* heranziehen.

Die Eigenschaft, dass die Projektion von σ_ν aus der Elementseite $E_1 E_2 \dots E_\nu$ die Elementseite $E_{\nu+1} E_{\nu+2} \dots E_{n+1}$ mit dem Grade 1 bedeckt, lässt sich nämlich von ν auf $\nu + 1$ ausdehnen, indem wir zunächst aus ihr folgern, dass die Projektion des in der Elementseite $E_1 \dots E_\nu E_{\nu+2} \dots E_{n+1}$ liegenden Teiles der Grenze von σ_ν aus der Elementseite $E_1 \dots E_\nu$ oder aus der Elementseite $E_1 \dots E_{\nu+1}$ die Elementseite $E_{\nu+2} \dots E_{n+1}$ mit dem Grade 1 bedeckt, und sodann σ_ν, indem wir jedesmal ein einziges seiner Grundsimplexe tilgen, stückweise auf γ_ν reduzieren, wobei der in der Elementseite $E_1 \dots E_\nu E_{\nu+2} \dots E_{n+1}$ liegende Teil der Grenze von σ_ν schrittweise in $\sigma_{\nu+1}$ übergeht und der entsprechende Projektionsgrad auf $E_{\nu+2} \dots E_{n+1}$ sich nicht ändern kann. Weil mithin jedes σ_ν ($\nu = 1, 2, 3, \dots n$) sich mit dem Grade 1 auf eine $(n-\nu)$-dimensionale Seite von σ projiziert, so kann keines der σ_ν sich auf Null reduzieren. W. z. b. w.

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menta Mathematicae, Bd. 2 (1921), S. 256—285 ist Herr Lebesgue auf den Gegenstand zurückgekommen und hat er einen stichhaltigen Beweis des Hilfssatzes gegeben, der, was den Kern betrifft, mit meinem obigen Beweise von 1913 übereinstimmt, davon aber durch eine unnötig verwickelte Darstellung der Einzelheiten abweicht.

[20]) Vgl. daselbst S. 105.

Mathematics. — "*Ueber Invarianten von Bilinearformen*". Von Prof. R. WEITZENBÖCK. (Mitgeteilt von Prof. L. E. J. BROUWER).

(Communicated at the meeting of November 24, 1923).

In der Theorie der endlichen diskreten Gruppen linearer Substitutionen besteht der Satz[1]: Notwendig und hinreichend für die Aequivalenz zweier Gruppen ist die Gleichheit ihrer Charaktersysteme. Von diesem Satze wird hier ein neuer Beweis gegeben, der die Theorie der affinen Invarianten derjenigen Bilinearformen benutzt, die den einzelnen Substitutionen einer Gruppe Γ zugeordnet sind. Im Besonderen wollen wir zeigen, dass die einzigen Invarianten dieser Bilinearformen die Charaktere der Substitutionen von Γ sind.

§ 1. *Bezeichnungen.*

Es sei $\Gamma = E, A, B, \ldots$ eine endliche Gruppe der Ordnung μ von n-ären linear-homogenen Substitutionen

$$(A) \quad \bar{x}_i = a_i^1 x_1 + a_i^2 x_2 + \ldots\ldots + a_i^n x_n \quad (i = 1, 2, \ldots, n). \quad (1)$$

E sei die Einheitssubstitution mit $e_i^i = 1$, $e_i^k = 0$ $(i \neq k)$; $a = |a_i^k|$ sei die Determinante der Matrix $\|a_i^k\|$ von A. $a, b, \ldots$ sind μ-te Einheitswurzeln.

Statt (1) schreiben wir auch kürzer

$$(A) \quad \bar{x}_i = a_i^\lambda x_\lambda, \quad \ldots\ldots\ldots\ldots \quad (2)$$

oder auch, symbolisch, für $a_i^k = a_i a'_k$ setzend:

$$\bar{x}_i = a_i (a' x). \quad \ldots\ldots\ldots \quad (3)$$

Der Substitution A ist zugeordnet die n-äre Bilinearform

$$L_A = a_i^k x_k u^i = (a' x)(a u'), \qquad L_E = x_i u^i = (u' x). \quad \ldots \quad (4)$$

Deren einfachste affine Invariante

$$\chi(A) = \sum_i a_i^i = (a' a) = a_1^1 + a_2^2 + \ldots\ldots + a_n^n. \quad \ldots \quad (5)$$

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[1]) Vgl. z. B. H. F. BLICHFELD. Finite Collineation Groups, Chicago (1917), p. 129 oder: A. SPEISER. Theorie der Gruppen von endlicher Ordnung, Berlin (1923), p. 116.

heisst der Charakter von A. $\chi(E) = n$, $\chi(A)$, $\chi(B), \ldots$ bilden das Charaktersystem der Gruppe Γ.

Die zu A *inverse* Substitution A^{-1} erhält man durch Auflösung von (2) nach den x_i Der durch a dividierte Minor von a_i^k in a werde mit A_k^i (Vertauschung der Indexstellung!) bezeichnet. Dann ist

$$(A^{-1}) \quad x_i = A_i^\lambda \bar{x}_\lambda \qquad [A_i^\lambda a_\lambda^\varkappa = e_i^\varkappa \ , \ A_\lambda^i a_\varkappa^\lambda = e_\varkappa^i]. \quad . \quad . \quad (6)$$

Die zu A *transponierte* Substitution A' ist dargestellt durch

$$(A') \quad w^i = a_\lambda^i \bar{u}^\lambda; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

und deren inverse $A_t = (A')^{-1}$ wird gegeben durch:

$$(A_t) \quad \bar{u}^i = A_\lambda^i u^\lambda \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

A_t heisst die zu A *kontragrediente* (oder adjungierte) Substitution. Nach (6) ist dann:

$$A_t = (A^{-1})' = (A')^{-1} \quad , \quad (A_t)_t = A \quad . \quad . \quad . \quad (9)$$

Die Veränderlichen x_i and u^i sind kontragredient zueinander. Die mit Γ homomorphe Gruppe $\Gamma_t = E, A_t B_t, \ldots$ heisst die zu Γ kontragrediente (oder adjungierte) Substitutionsgruppe Γ_t. Es lässt sich leicht zeigen[1]), dass der Charakter $\chi(A_t)$ die zu $\chi(A)$ konjugiert-komplexe Zahl ist.

Analog zu (8) und (4) ist

$$L_{A_t} = A_k^i x_i \, u^k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

die zu L_A kontragrediente Bilinearform; symbolisch wird sie, wenn $a_1, a_2, \ldots$ und $a'_1, a'_2, \ldots$ äequivalente Symbolreihen darstellen, gegeben durch

$$L_{A_t} = \frac{1}{a} \cdot \frac{1}{(n-1)!} \cdot (a_1 a_2 \ldots a_{n-1} x)(a'_1 a'_2 \ldots a'_{n-1} u') \quad . \quad (11)$$

Die Determinante a ist symbolisch gegeben durch:

$$a = \frac{1}{n!} (a_1 a_2 \ldots a_n)(a'_1 a'_2 \ldots a'_n) \quad . \quad . \quad . \quad . \quad (12)$$

Dem Produkte $AB = C$ zweier Substitutionen A und B ist zugeordnet die Bilinearform

$$L_{AB} = a_i^\lambda b_\lambda^k x_k u^i = (u' a)(a' b)(b' x) = c_i^k x_k u^i, \quad . \quad . \quad (13)$$

während der Substitution BA zugeordnet ist:

$$L_{BA} = b_i^\lambda a_\lambda^k x_k u^i \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

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[1]) Speiser, l.c. p. 110.

Wegen der Gruppennatur führt jede Zusammensetzung der Gestalt

$$a_i^{\lambda} b_{\mu}^{\mu} c_{\nu}^{\nu} \ldots g_{\rho}^{k} = h_i^k \quad \ldots \quad (15)$$

wieder auf eine Substitution H zurück.

§ 2. *Das volle Komitantensystem.*

Wir konstruieren jetzt ein volles System von affinen Komitanten der μ Bilinearformen (4) mit der Einschränkung, dass wir neben den Koëffizienten dieser Bilinearformen nur noch eine Reihe x und eine Reihe u zulassen.

Zur Verfügung stehen dann die Reihen

$$a_1, a_2, \ldots, x \quad \text{und} \quad a'_1, a'_2, \ldots, u'. \quad \ldots \quad (16)$$

Dabei soll $(a_i)_r\,(a'_i)_s$ gleich dem Koeffizienten a_r^s irgend einer der Formen (4) sein. Aus (16) bilden wir: 1. Faktoren *zweiter* Art der Gestalt:

$$\varphi_1 = (a_1 a_2 .. a_n),\ \varphi_2 = (a_1 a_2 .. a_{n-1} x);\ \psi_1 = (a'_1 a'_2 .. a'_n),\ \psi_2 = (a'_1 a'_2 .. a'_{n-1} u'); \quad (17)$$

2. Faktoren *erster* Art der Gestalt:

$$f_1 = (a_i a'_k),\ f_2 = (a\,u'),\ f_3 = (a' x),\ f_4 = (u' x). \quad (18)$$

Jede affine Komitante I ist ein Produkt dieser Faktoren. Wir können annehmen, dass in I nicht φ und ψ gleichzeitig auftreten, da das Produkt eines φ und eines ψ durch Faktoren f ausdrückbar ist wegen

$$(a_1 a_2 \ldots a_n)(a'_1 a'_2 \ldots a'_n) = \begin{vmatrix} (a_1 a'_1) & \ldots & (a_1 a'_n) \\ \ldots & \ldots & \ldots \\ (a_n a'_1) & \ldots & (a_n a'_n) \end{vmatrix} \quad (19)$$

Es enthalte nun I einen Faktor $\varphi: I = (a_1 a_2 a_3 \ldots) I'$. In I' suchen wir a'_1 auf, das in einem f stecken muss: $I = (a_1 a_2 a_3 \ldots)(a'_1 a_r) I''$. In I'' suchen wir a'_r auf, das wieder in einem f steckt:

$$I = (a_1 a_2 a_3 \ldots)(a'_1 a_r)(a'_r a_s) \ldots.$$

Dies geht so fort, bis die Kette $(a'_1 a_r)(a'_r a_s)(a'_s a_t) \ldots$ mit einem Gliede $(a' x)$ abbricht. Mit $a_2, a_3, \ldots$ machen wir es analog und erhalten für I im Falle der Anwesenheit eines Faktors φ_1 oder φ_2 die Gestalt:

$$I = (a_1 a_2 a_3 \ldots)\underbrace{(a'_1 a_r) \ldots (a'_\rho x)}_{K_1} \cdot \underbrace{(a'_2 a_s) \ldots (a'_\sigma x)}_{K_2} \ldots \quad (20)$$

Die hier mit $K_1, K_2, \ldots$ angedeuteten Ketten können dabei beliebig lang sein.

Eine ganz analoge Gestalt bekommt I bei Anwesenheit von ψ_1 oder ψ_2, nur dass dann die entsprechenden Ketten mit u' endigen. Invarianten (ohne x oder u') erhält man sonach hier nicht.

Es wäre nun nicht schwer bei allgemeinen Bilinearformen die Bildungen (20) auf gewisse einfache Gestalten zu reduzieren. Man kann z. B. bei den Ketten K die Gliederzahl stets $\leqq n-1$ voraussetzen. Doch haben wir dies hier nicht nötig, da unsere Substitutionen $A, B, \ldots$ eine endliche diskrete Gruppe bilden, wodurch sich die Sache sehr vereinfacht. Jede Kette führt nämlich nach (15) wieder auf ein einziges $h_i h'_i$ zurück und diese Reihen müssen untereinander verschieden sein, wenn $J \equiv\!|\!\equiv 0$ ist. Wir erhalten somit im Falle $\mu \geqq n$ je $\binom{\mu}{n}$ Komitanten der zwei Typen:

$$I = (a_{i_1}\, a_{i_2} \ldots . a_{i_n})\,(a'_{i_1}\, x)\,(a'_{i_2}\, x) \ldots \ldots (a'_{i_n} x) \quad . \quad . \quad (21)$$
$$(i_r \neq i_s)$$

$$I' = (a'_{i_1}\, a'_{i_2} \ldots . a'_{i_n})\,(a_{i_1}\, u')\,(a_{i_2}\, u') \ldots \ldots (a_{i_n}\, u') \quad . \quad . \quad (22)$$

Hier sind auch die Komitenten mit φ_2 und ψ_2 mitaufgezählt, denn es ist z. B. bei φ_2 eines der $a_i a'_k$ gleich $e_i^k = e_i e'_k$.

Wir kommen zu Faktoren erster Art. f_4 ist bereits eine Komitante, nämlich die Bilinearform L_E. Bei den übrigen Faktoren f bilden wir Ketten, von denen zweierlei Typen möglich sind:

$$T_1 \ldots . (x a'_i)\,(a_i a'_k) \ldots . (a_r a'_s)\,(a_s u')$$
$$T_2 \ldots . (a a'_i)\,(a_i a'_k) \ldots . (a_r a'_s)\,(a_s a').$$

Auch diese Ketten reduzieren sich wegen (15) auf einfachste Formen: T_1 auf die Bilinearformen L_A, $L_B, \ldots$ selbst, T_2 auf die Charaktere $\chi(A) = (a a')$. Diese Charaktere sind somit die einzigen affinen Invarianten der Bilinearformen L. Gleichheit der Charaktere bei entsprechenden Substitutionen homomorpher Gruppen Γ und Γ' bedeutet also Gleichheit der affinen Invarianten der entsprechenden Bilinearformen L und L'. Der Homomorphismus garantiert zwischen den Koeffizienten der L dieselben affin-invarianten Gleichungen wie zwischen den Koeffizienten der L'. Die L sind also bezgl. affiner Transformationen den L' äquivalent.

Physics. — *"The Influence of Rotation on the Sensitiveness and the Accuracy of a Pressure Balance."* (Twelfth communication of results obtained in researches made by the aid of the VAN DER WAALS fund). By A. MICHELS. (Communicated by Prof. P. ZEEMAN).

(Communicated at the meeting of October 27, 1923).

For the accurate measurement of great pressures methods are now of general application, based on the use of the so-called Amagat cylinder. In all these methods the force is studied exerted by a liquid under pressure on a piston of known diameter. The elaboration of this fundamental idea has given rise to different types of pressure balances, as those of WAGNER, STÜCKRADT, SCHÄFFER und BUDENBERG, and HOLBORN [1]).

In order to reach an accuracy as great as possible it is necessary to reduce the frictional forces between the piston and the wall of the hole to a minimum. In this respect WIEBE already obtained good results by tapping the wall of his apparatus with a hammer. Of late a rotation of the piston has pretty generally been applied, though HOLBORN [2]) considers a movement to and fro preferable.

The causes why these operations have such an influence, are only imperfectly known as yet. KLEIN (loc. cit.) tries, indeed, to give a solution of the effect of rotation, but does not succeed.

The purpose of this investigation is to find a solution, and at the same time to determine the circumstances under which the greatest effect is reached.

As there is no room here for an extensive discussion of our results, we shall restrict ourselves in what follows to a brief communication, referring for a fuller treatment to "Annalen der Physik" Bd. 72, 1923, p. 285—320.

It was tried to work theoretically in the direction indicated by the recent theory of bearings lubricated all round [3]). For when the piston revolves in a cylindrical hole, liquid being continually supplied from below, there must certainly be an analogy between the influences of friction to which our piston is subjected and those exerted on an ordinary axle resting in a bearing block.

[1]) I refer for the different types to KLEIN. G. Untersuchung und Kritik von Hochdruckmesser Diss. Berlin 1909.

[2]) Ann. d. Physik 1915, p. 1087.

[3]) SOMMERFELD. Zeitschr. für Math. und Physik 1904, GÜMBEL. Das problem der Lagerreibung Jahresb. d. Schiffbautechn. Gesellsch. 1917.

Undoubtedly there are also points of difference, which must be chiefly owing to this that in our case the so-called bearing-pressure is wanting on account of the vertical position of the piston. Application of the theory taught that if the peripherical speed is sufficient, a liquid layer will be formed everywhere between piston and hole-wall. The number of revolutions at which this takes place, will be called the critical value of the revolutions ω_c. It is dependent on the viscosity of the liquid chosen. In the absence of any metal contact also the axial friction would be a liquid friction above this value of revolutions.

In order to test the validity of this theory the pressure balance of the VAN DER WAALS fund which was at our disposal, was modified in such a way that it had a driving apparatus that could be regulated mechanically.

This alteration was made by the instrument-maker of the laboratory, Mr. J. WASSENAAR.

Characteristic of a liquid friction is its proportionality with the velocity. When a definite initial value of revolutions Ω is given to the piston, after which the motor is cut out, the motion will be retarded, and the angle $\varkappa$ passed over in the time t, will get a value of

$$\alpha = \frac{\Omega}{A}\left(1 - e^{-At}\right)$$

in which A is a constant. As soon, as the value of revolutions descends below the critical value however, there is metal contact, and the image of the motion changes.

In this way the course is examined all over the measuring scope of the pressure balance, and agreement was found between experiment and theory. As was to be expected, the critical value of the revolutions then appeared to be dependent on the temperature, as this influences the viscosity, but independent of the load.

An electrical determination shows the validity of the suppositions still more clearly. For, when the electrical resistance between axle and wall was measured, it appeared to be about 700 Ohms above a definite number of revolutions, being reduced pretty suddenly to 0.2 Ohm on diminution of the velocity. In these values the resistance of the conducting wires is included.

Conclusion. For a favourable use of the pressure balance experiments should always be made above the critical value of revolutions. This value can be determined experimentally for every liquid and temperature.

Anatomy. — *"The Forebrain of Apteryx Australis"*. By JOHN I. HUNTER, M. B. Ch. M. (Sydney). (From the Central Institute of Brain Research, Amsterdam). (Communicated by Prof. L. BOLK).

(Communicated at the meeting of December 29, 1923).

I. *General Features.*

An examination of the external form of the brain of the New Zealand kiwi (*Apteryx australis*) reveals the presence of distinct differences from the usual condition exhibited by the avian brain. The general shape of the cerebral hemispheres is peculiar in that the frontal extremities are somewhat more pointed than usual, and the lateral surface proceeds backwards by a gentle convexity to the posterior extremity.

The characteristic subdivision of the cerebral hemisphere of birds into a *pars medialis* and *pars lateralis*, of which the *pars lateralis*, may enwrap the *pars medialis* to form the frontal pole, is not visible in this specimen (fig. I). For the *pars medialis* (sagittal-wulst of EDINGER, WALLENBERG and HOLMES, 1903) is not indicated, though there is an ill-defined bulging on the postero-medial part of the dorsal surface of this hemisphere. In consequence of this the *vallecula*,

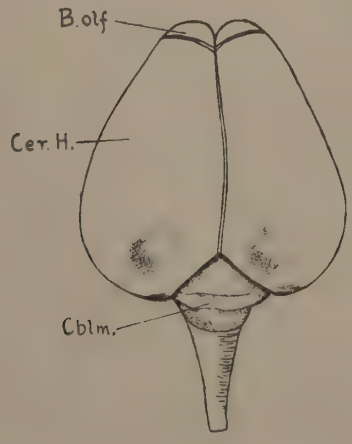

Fig. I.

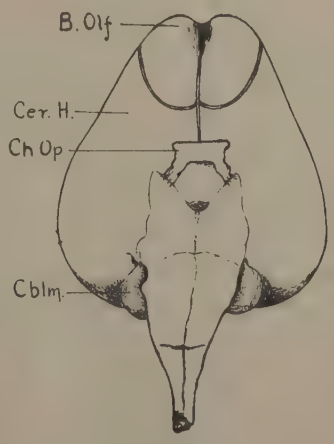

Fig. II.

which usually limits the *pars medialis* laterally, is not conspicuous. (cf. fig. I with fig. 535—537 ARIËNS KAPPERS, 1291. Vide also OWEN, 1872, p. 382).

Another important feature of the brain is the presence of two large olfactory lobes (fig. II). These project for a short distance beyond the anterior extremity of the fore-brain (fig. I and fig. II). Extending posteriorly they receive a very wide attachment to the ventral aspect of the frontal region of their respective hemispheres (fig. II). In marked contrast with this unique degree of development amongst *Aves* of the olfactory lobes, the visual apparatus is very poorly developed compared with a typical avian brain, as is indicated by the smallness of the optic nerves, chiasma, tracts and lobes (fig. II). This enhanced importance of the smell centres and associated reduction in the importance of the visual connections, combined with the presence of an apparently simpler hemisphere than is usually the case in *Aves*, suggest the conclusion that the brain of the kiwi is a comparatively simple and primitive type of avian brain, (cf. OWEN 1841, p. 287). For these reasons Dr. C. U. ARIËNS KAPPERS kindly suggested that I should undertake the investigation of this brain. It is a pleasure to express to him my great indebtedness on this account, and because of the assistance he afforded me in carrying out the comparative investigation necessary to elucidate the somewhat unusual features of the specimen.

II. *Technique.*

The material consists of a transverse series of sections of a single brain. Alternate sections were stained by the VAN GIESON method; the series remaining was treated by the Weigert-Pal-para-carmine method described by ARIËNS KAPPERS and KETJEN (1911). Unfortunately the specimen was in alcohol when received by Professor ELLIOT SMITH from the Zoological Gardens, London, who kindly transmitted it to the Central Institute of Brain Research, Amsterdam, after transference to formalin. The brain was evidently in a bad state of preservation before being hardened. In consequence, the condition of the sections is not good and a final analysis of the cell masses and their fibre connexions is not possible. However many features are so clearly defined that a description of them may be entered upon with confidence. To control the topographical description of the various parts a wax plate-ceresine reconstruction twelve and a half times the size of the original, was prepared. (cf. ARIËNS KAPPERS 1915).

III. *Description of the sections.*[1])

a. Connections of the olfactory nerves.

As already mentioned the olfactory bulbs and lobes are conspicuous structures bilaterally represented. The most frontal sections show a bulbar formation which is arranged in a circular manner (Fig. III) though no extension of the ventricle (rhinocoele) is visible in this region[2]). The fila olfactoria, glomeruli and mitral cells are of the usual structure (cf. EDINGER, WALLENBERG, HOLMES, 1903, p. 403) and call for no special description. The two separate formations right and left, are distinctly seen throughout (Fig. III and IV). TURNER (1891, p. 43) and S. P. GAGE (1893, p. 197) refer to the degree of

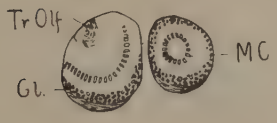

Fig. III

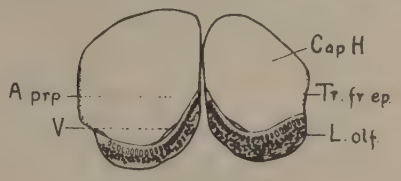

Fig. IV

diminution in importance of the olfactory connexions in *Aves*, culminating in the concrescence of two small lobes in some higher forms, as an index of the stage of organisation attained by the brain. The *lobus olfactorius* is spread out upon the ventral aspect of the anterior part of the cerebral hemisphere and is crescentic in cross section (Fig. IV). The second relay olfactory fibres form a distinct tract in the most dorsal lamina of this structure immediately ventral to a small forward prolongation of the lateral ventricle which becomes visible in this region (Fig. IV). These fibres end in the *cortex lobi olfactorii* or *area praepiriformis* of BRODMANN (cf. ROSE, 1914, p. 338). The position of this area immediately dorsal and medial to this ventricular extension can be located in the sections (Fig. IV), though its cell structure is not clearly distinguishable (cf. ROSE, op. cit. p. 339, Taf. III Fig. 8, 9, 61, Taf. I, Fig. 13).

Dorsal and caudal to the *area praepiriformis* the frontal portion of the *septum* is an extremely thin double lamina. (It is shown somewhat crumpled in the diagrams; cf. Fig. V). Somewhat more caudally frontal to and in the region of the anterior commissure

[1]) The sections corresponding to the figures are as follows: III, 21; IV, 58; V, 179; VI, 204; VII, 208; VIII, 212; IX, 235; X, 245; XI, 262; XII, 283 XIII, 293; XIV, 303.

[2]) For the lettering used in the figures see pages 822 and 823.

the *nucleus lateralis septi* is a conspicuous structure. The *nucleus medialis septi* is also visible. The *zona gliosa limitans* separates these nuclei from the Area 28 of Rose (1914) which is well defined. The medial limit of this zone is indicated by the *fissura septo-pallialis*. On the ventricular side of the septum the lateral limit of the zone is marked by the *fissura limitans hippocampi*. Dorsal to the Area 28 slight indentations laterally and medially serve to mark off this area from the cortex.

As the secondary olfactory fibres disappear posteriorly they are replaced by a great fibre field which extends completely across the ventral portion of the hemisphere. Fig. IV shows that the major portion of the hemisphere in this region consists of the corpus striatum (*caput hyperstriati*). The fibres first become distinct on the

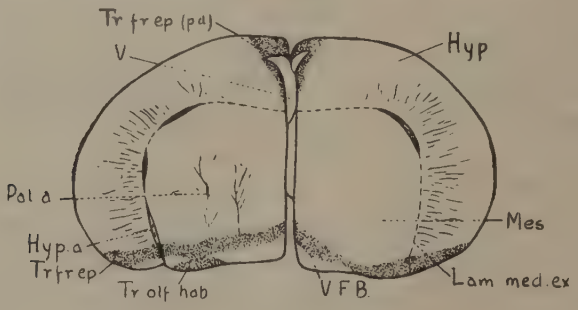

Fig. V

lateral surface of this structure but are soon seen to be arising from the whole ventral region of the hemisphere extending to the medial wall. When this extent has been attained (Fig. V), it becomes obvious, as the connexions of the fibres also show, that there are three main elements in this fibre field.

The most medial bundle arises from the region of the *area praepiriformis* and septum. It forms a conspicuous tract which separates from the remaining fibres in order to enter the diencephalon medial to the *tractus septo-mesencephalicus* (Fig. VI). This tract is the ventral forebrain bundle (*basales Riechbündel*). In the diencephalon it takes up its position lateral to the third ventricle (Fig. VIII) and extends backwards as far as the *nucleus oculomotorius*, (Fig. X). as was shown by Jelgersma (1896).

Lateral to the ventral forebrain bundle and ventral to the main field of fibres a second conspicuous myelinated tract is to be seen. It lies ventral to the base of the mesostriatum and so occupies a superficial position (Fig. V). When the ventral forebrain bundle enters the diencephalon it comes to lie more medially (Fig. VI),

and later bridges across the floor of the fissure separating the telencephalon from the diencephalon (Fig. XI). In this situation it forms a conspicuous oval bundle which is visible in the sections to the

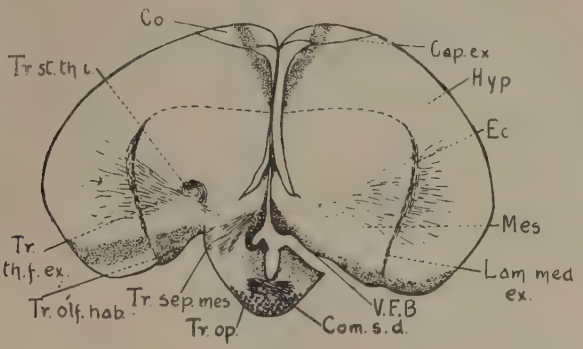

Fig. VI

naked eye. This tract is the principal constituent of the *taenia thalami* representing the element called *olfacto-habenular* by Edinger and Wallenberg (1899), p. 251). A similar tract is figured by Edinger, Wallenberg and Holmes (Taf. V) and Schroeder (Fig. 47) but in these cases it is of considerably smaller dimensions than that attained in the kiwi. The condition of preservation of the specimen prevents the identification of a *nucleus taeniae*. When traced medially the tract passes to the lateral aspect of the ganglion habenulae and gradually ends in it. Many fibres cross the median plane returning apparently to the forebrain on the other side forming a very conspicuous *commissura telencephali superior* (Fig. XII). In reviewing

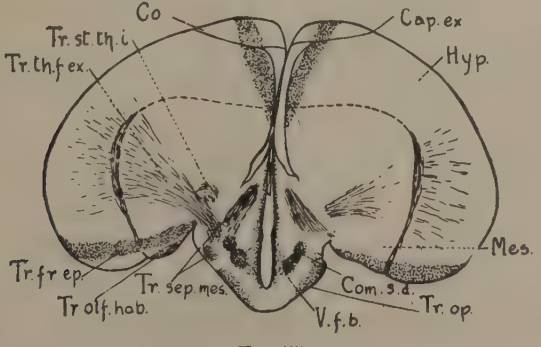

Fig. VII

the fibre tracts of the avian brain, Ariëns Kappers (1921, p. 1046) considers that the presence of this commissure in birds is questionable, though it is clearly present in all animals (cf. Varanus sal-

vator, as figured by DE LANGE, 1911, fig. 25, where the *commissura telencephali superior* is shown). Co-existing with this commissure in Varanus is a well-marked *commissura pallii posterior* or *commissura aberrans* of ELLIOT SMITH (DE LANGE, op. cit. fig. 21) which is absent in *Aves*. The relations of the tract forming the commissura in the kiwi are so precise that there can be no doubt that the commissure present here is not the *commissura pallii posterior*, but the *commissura telencephali superior* (cf. ARIËNS KAPPERS, 1921, p. 797 and footnote; p. 1034) I am unable to recognise the *commissura pallii* (cf. SCHROEDER fig. 42) in the sections under examination.

The remaining fibres of the ventrally situated fibre field constitute

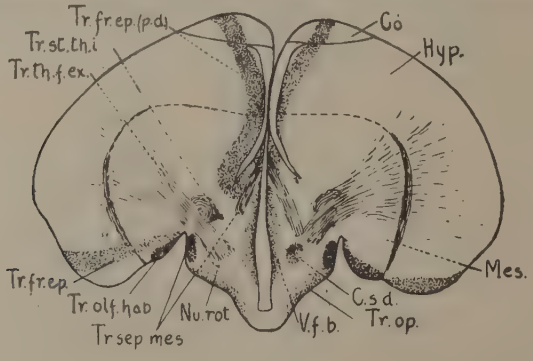

Fig. VIII

the fronto-occipital (fronto-epistriatic or lobo-epistriatic) tract: This arises over a wide area of the ventral aspect of the hemisphere as described by EDINGER, WALLENBERG and HOLMES (1203), (p. 381 and Fig. 5*b*) In the kiwi this tract can be traced to the posterior end of the corpus striatum where its fibres terminate. It is here seen to be augmented considerably by the addition of another great bundle of fibres, the connexions of which are also fronto-occipital. This tract first becomes distinct immediately dorsal to the slight lateral extension of the dorsal part of the lateral ventricle in the cortex region (Fig. V), but it soon appears on the ventricular aspect of the corpus striatum also and increases in size until it is a very extensive tract. It contributes a few fibres to the *commissura anterior* (Fig. IX) and then becomes merged with the fronto-occipital bundle already described to form a conspicuous tract which is oval in cross section (Fig. XIV). Similar fibres to these are described by EDINGER, WALLENBERG and HOLMES (op. cit. p. 383) who figure the fronto-occipital bundle divided into a dorsal and ventral part in the sparrow (Taf. II, Fig. 4). SCHROEDER, (1911, p. 145) in his excellent

account of the order of myelinisation of the fibre tracts in the chick, demonstrates the presence of a band of fibres on the ventricular aspect of the dorso-occipital part of the corpus striatum. Some of these fibres in the kiwi enter into the formation of the inter-epistriatic commissure. It is probable that these commissural fibres are comparable to the *fibrae marginales* found on the ventricular aspect of the striatum of Varanus and crossing to the opposite side in the *commissura anterior* (cf. DE LANGE, op. cit. Fg. 19, 20). The further description of the inter-epistriatic commissure will be deferred until the discussion of the subdivisions of the corpus striatum is undertaken.

b. The Corpus Striatum.

Notwithstanding the unusual external features of the brain of *Apteryx* to which reference has already been made the outstanding features of the sections are definitely avian. In 1891 Professor T. J. PARKER observed that his investigations of the development of the brain of the kiwi though very imperfect owing to lack of material "prove conclusively what might have been inferred from adult anatomy, that the brain of *Apteryx* is simply a typical avian encephalon with reduced optic lobes." (p. 107).

As is usual in *Aves*, the forebrain of the kiwi consists, for its greater part, of the corpus striatum. This body appears on each side as a great ventricular bulging. Frontally it forms the frontal pole of the hemisphere. Caudally its posterior extremity projects freely into the ventricle in close proximity to the hinder pole of the hemisphere. In the more frontal sections (Fig. V—VIII) the lateral ventricles form two vertical slits separated from one another by the two thin laminae constituting the septum and the corpora striata form the vertical lateral boundaries of the ventricles in this region.

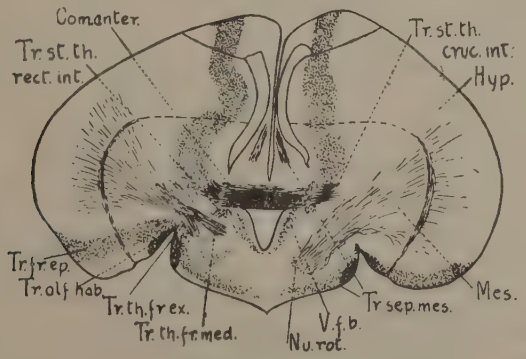

Fig. IX

Further caudally the ventricles increase in size being less reduced by the encroachment of the corpora striata laterally (Fig. IX—XIV).

The criterion of cell structure (cf. Rose, 1914) cannot be employed in analysing the constitution of the corpus striatum in this instance on account of the poor state of preservation of the specimen. For this purpose it becomes necessary to rely upon fibre connections and topographical relations in differentiating its various parts. In naming these the nomenclature of Edinger (1896) will be followed.

Fortunately the *nucleus entopeduncularе* can be recognised at the junction of the telencephalon and diencephalon. Surrounding this nucleus is a large-celled area forming the *nucleus basalis* constituting the *palaeostriatum primitivum* (Ariëns Kappers, 1908; 1922). This is surrounded by a larger part of the striatum which is an extension of the palaeostriatum — the mesostriatum or *palaeostriatum augmentatum*, (Ariëns Kappers, 1922, p. 140). The *lamina medullaris*

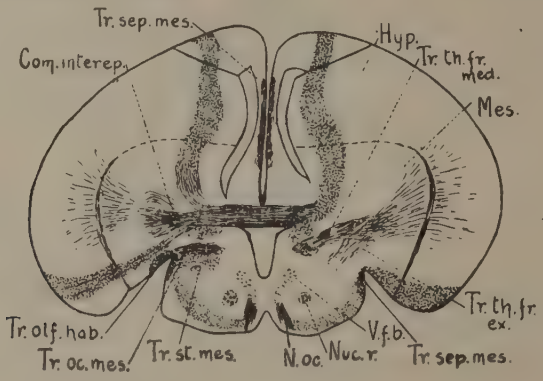

Fig. X

ventralis (*lamina medullaris interna* of Kappers, op. cit.) which separates these two parts from one another is not to be seen in the sections at hand.

If the sections showing these areas be examined (Fig. VI—VII) it is very evident that dorso-lateral to the mesostriatum there lies a mass of considerable size constituting the hyperstriatum. Its extent is as follows. Dorsally it is separated from the cortex by an ill defined lamina of fibres (*capsula externa* of Edinger, Wallenberg and Holmes, op. cit. p. 365; Taf. V). Traced laterally and ventrally it becomes continuous with the external or pallial surface. This mass is separated from the mesostriatum by a layer of fibres which constitute the *lamina medullaris dorsalis* (cf. Edinger, Wallenberg and Holmes op. cit. p. 390; Schroeder, op. cit. p. 141). This subdivision is exceptionally clear in *Apterix*. This is partly due to the

fact that this lamina is richly provided with blood-vessels, a point to which I shall return later. It extends from the ventricular surface

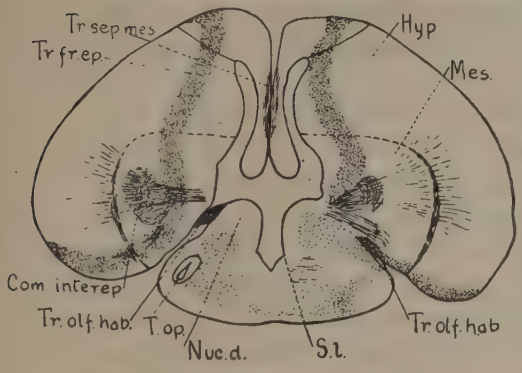

Fig XI

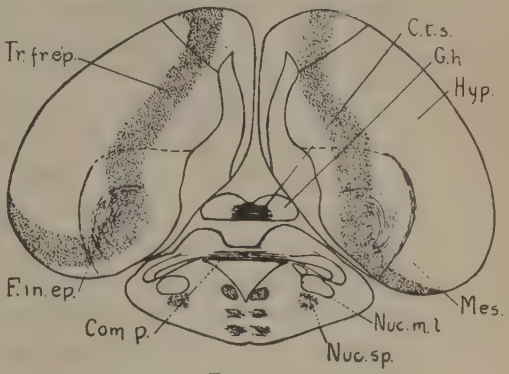

Fig XII

of the corpus striatum medially to an external groove marking the interval between the hyperstriatum and mesostriatum ventrally. Therefore this lamina separates the two parts not only dorsally but laterally. For this reason as Ariëns Kappers has suggested (1922) it is preferable to employ the term *lamina medullaris externa* in referring to this fibre zone. The fibres constituting it, which are connected on the one hand with the thalamus (vide infra), radiate laterally between columns of cells of the hyperstriatum. This confers a striated appearance upon this structure especially in its ventro-lateral part.

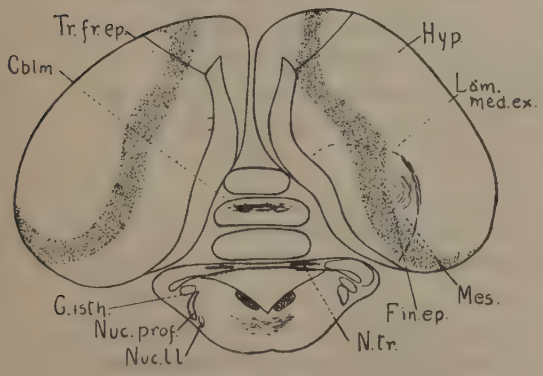

Fig XIII

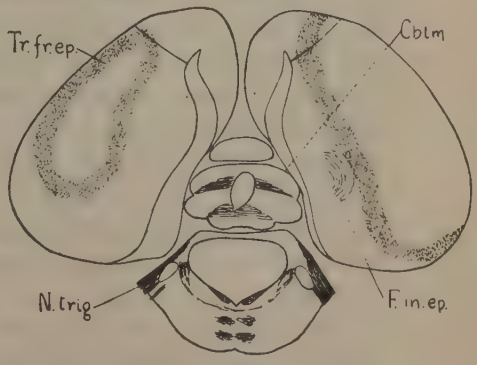

Fig XIV

The ectostriatum which lies between the mesostriatum and hyperstriatum and is recognisable by the naked-eye on account of its infiltration by fibres, and microscopically by the presence of large

cells, is not conspicuous in this specimen although the thickened fibre zone lateral to the external medullary lamina (Fig. VI) is comparable to the area figured by Edinger, Wallenberg and Holmes (op. cit) in *Lothrix lateus* (Taf. III, fig. 5 and 6) and *Sylvia hortensis* (Taf. IV fig. 6).

In the sections under examination the hyperstriatum shows no clear sign of subdivision. In most birds the *lamina medullaris hyperstriati* divides the hyperstriatum into the *hyperstriatum superius* and *hyperstriatum inferius* of Ariëns Kappers (1922). As this author rightly points out (op. cit. note 1, p. 23) Parker's figures, in the work already mentioned on the development of the brain of the kiwi, show only two intraventricular primordia which probably represent the palaeostriatum and hyperstriatum inferius, the hyperstriatum superius being apparently absent, (Parker 1891, Plate 19, Fig. 304). This point however needs re-investigation upon material in a better state of preservation than that at present available.

Frontally the hyperstriatum covers the frontal pole of the mesostriatum and forms the frontal extremity of the cerebral hemisphere (*caput hyperstriati*, Fig. IV).

The caudal part of the corpus striatum receives the fronto-epistriatic tracts (Fig. XIV). This region constitutes the secondary epistriatum or archistriatum (Ariëns Kappers 1908). It is the area called epistriatum in the memoir of Edinger, Wallenberg and Holmes. The *fissura strio-archistriatica* is not visible in the bird's brain. The archistriatum is connected to its fellow of the opposite side by a great strand of fibres (*commissura interepistriatica*, Edinger). This bundle forms the main constituent of the *commissura anterior* (vide Fig. X) which is large and conspicuous in this brain (Fig. IX—X). Some of these fibres accompany the fronto-epistriatic tracts; the majority form a distinct fibre field in the ventral part of the mesostriatum (Fig. IX—XIII).

c. The significance of the blood vessels accompanying the lamina medullaris externa.

In his recent work on the morphology of the corpus striatum, Elliot Smith (1919*b*) has emphasised his contention that the great ventricular eminence which forms such a conspicuous feature of the cerebral hemisphere of *Sphenodon* is pallial in origin. He has introduced the term hypopallium to designate this structure because "it is pallial in origin; it lies below the main portion of the pallium which forms the roof of the hemisphere; and morphologically and

functionally it is analogous to but upon a lower plane of usefulness than the neopallium", (op. cit. p. 272). He has established the truth of this statement for *Reptilia* in general by reference to examples of the *Ophidia*, *Lacertilia*, and *Chelonia*. Moreover, he points out, that the evidence now goes to show that every mammalian brain passes through a stage of development in which the corpus striatum is clearly divisible into hypopallium and palaeostriatum. Subsequent development shows that the hypopallium in man gives rise to the putamen and most of the caudate nucleus (together constituting the neostriatum of Ariëns Kappers), the claustrum, and the hypopallial element of the *nucleus amygdalae*. The palaeostriatum forms the *globus pallidus* and according to Elliot Smith a small part of the caudate nucleus (op. cit. p. 291; vide however, Ariëns Kappers 1922, p. 153).

In *Sphenodon* the boundary line between the hypopallium and palaeostriatum is indicated by the course of large arterial vessels and emerging veins, the former constituting the lateral striate artery of reptiles, (Elliot Smith op. cit. p. 272).

Shellshear (1920) has identified this artery in the adult human brain immediately lateral to the palaeostriatal area and has called it the claustral (or hypopallial) artery. It seems that, in conformity with the phylogenetic and ontogenetic history of the pallial origin of the hypopallium, pallial vessels have become hypertrophied at the site of intilting of the pallium to supply this new pallial development. Deeply penetrating into the hemisphere they form in man a lateral group of the antero-lateral basal vessels of the middle cerebral artery.

The vascular supply of the corpus striatum of *Apteryx* presents some remarkable features. In the first place a series of large vessels enter the base of the hemisphere in the *fissura ventralis* (*fissura limbica*, Edinger) and penetrate deeply into the corpus striatum (Fig. V). The course of these arteries follows very closely that of the external medullary lamina; in other words they form a clear line of seperation between the hyperstriatum laterally and the mesostriatum medially. This arrangement is constant even in the most posterior region in which the external medullary lamina can be identified and the separation of the mesostriatum from the hyperstriatum distinguished. Medial to this fissure many smaller vessels penetrate the corpus striatum in the region of the palaeostriatum and the blood supply is considerably from this source, (cf. Owen 1872, p. 381).

In contrast with this the blood supply entering the corpus striatum

on its lateral surface is very small. In this respect this great surface area which is formed by the hyperstriatum is sharply differentiated from the pallium which receives a relatively rich supply of vessels entering from the surface. The result of this arrangement is that the blood supply of the lateral part of the hyperstriatum is derived from a series of vessels penetrating deeply into the hemisphere along the line of the external medullary lamina and sending frequent branches laterally. Such a deep penetration of vessels demands an explanation which is to be sought on phylogenetic grounds. The explanation which suggests itself is that the hyperstriatal artery of this avian brain represents the lateral striate artery of reptiles and the hypopallial (claustral) artery of man. That, in effect it is a greatly hypertrophied vessel originally in series with the pallial arteries which are in the bird's brain mainly confined to the dorsal aspect, the lateral series having been greatly reduced in importance; and further that this hypertrophy has occurred because the hyperstriatum is pallial in origin. The hyperstriatum and the archistriatum together represent the hypopallium of reptiles and therefore also the hypopallial elements of the corpus striatum (hyperstriatum) and of the *nucleus amygdalae* (archistriatum) of the mammalian brain, (cf. Elliot Smith 1919a; Dart 1920).

The mesostriatum can be excluded from this complex on account of the difference in the origin of its blood supply alone. For the vessels situated more medially (palaeostriatal arteries) supply not only the basal nucleus or *palaeostriatum primitivum* (Ariëns Kappers 1922) but the extensive mesostriatum which surrounds it, (fig. 5). If the criterion of blood supply is to be applied (cf. Shellshear op. cit. p. 35) in this case, the mesostriatum must be regarded as an augmentative homology of the palaeostriatum so forming the *palaeostriatum augmentatum* of Ariëns Kappers (1922).

An examination of a series of sections of *Pratincola rubra* (figured by Ariëns Kappers in his text book, 1921) *Casuaris*, *Athene noctua*, *Paleornis*, *Ciconia alba*, reveals the fact that the same vascular arrangement as described in *Apteryx* holds for *Aves* in general. But in *Apteryx* the arrangement is displayed with the greatest clearness.

It follows from the above discussion that the *lamina medullaris externa* of birds is the line of separation of the neostriatum from the palaeostriatum and that the point where this lamina reaches the ventricle (e. g. fig. 6) represents the site of the *jissura neopalaeostriatica* which is clearly seen if embryonic stages of the chick's brain be studied, (cf. Ariëns Kappers, 1922, p. 140).

d. Ariëns Kappers' *Studies on the ontogeny of the corpus striatum of birds.*

In a recent paper Ariëns Kappers (1922) reported the results of his investigations upon the ontogenetic development of the different parts of the striatum complex in birds. He concludes that apart from the archistriatum "at least two chief divisions of the striatum may be distinguished: the palaeostriatum, which is enlarged to a *palaeostriatum augmentatum* (or meso-striatum) and which arises entirely from the base of the brain in front of the recessus praeopticus, and the hyperstriatum of which the upper part arises entirely from the mantle (*hyperstriatum superius*), while the underpart (*hyperstriatum inferius*), arises from the mantle (laterally) as well as from the base of the brain in front of the palaeostriatum. Both parts of the hyperstriatum thus show the fact, that intraventricular protrusions of striatal type may originate from the pallium as well as from the base of the brain, as I already pointed out for the primary epistriatum in bony fishes, and as was pointed out by Elliot Smith for the neostriatum of reptiles". (op. cit. p. 148).

The arrangement of the blood vessels in the adult bird's brain is in accord with these results based upon ontogenetic studies. Moreover the material employed demonstrates the fact that in the embryo the *lamina medullaris externa* "is a place of predilection for blood vessels", (op. cit. p. 146, cf. figs. 11, 12, 13).

e. Summary of the Fibre-Tracts of the Fore-Brain.

The following tracts have already been discussed.

1. Ventral forebrain bundle.
2. Olfacto-habenular tract.
3. Superior telencephalic commissure.
4. Pallial commissure.
5. Fronto-epistriatic tract.
6. Interepistriatic commissure.

Three bundles connect the forebrain with the mesencephalon.

1. *Tractus strio-mesencephalicus.* This tract, which connects the mesostriatum with the *nucleus spiriformis* (fig. XII), can be recognised in its course through brain stem (fig. X).

2. *Tractus occipito-mesencephalicus.* The occipito-mesencephalic tract takes origin in the archistriatum and ends in the *nucleus spiriformis* and neighbouring gray matter of the mesencephalon.

It enters the brain stem ventral to the anterior commissure arching

over the strio-thalamic and strio-mesencephalic bundles (Fig. X; cf. SCHROEDER fig. 42, 47).

3. *Tractus septo-mesencephalicus.* This tract forms a very conspicuous bundle in the kiwi. Arising from the cortex and septum (Fig. IX, X, XI), it passes forwards to turn laterally in front of the tractus thalamo-frontalis externus (Fig. VI). Trace dcaudally it occupies a superficial position in the brain stem (Fig. VIII). In this situation it may be traced as far as the *tectum opticum* (Fig. VIII, IX, X). The details of its connexions with the nucleus of the septo-mesencephalic tract, with the *tectum opticum*, the oculomotor nucleus and the caudal portion of the brain stem cannot be followed in the sections.

Tracts of considerable size connect the corpus striatum and diencephalon as follows.

1. *Tractus thalamo-frontalis externus.* This bundle originates from the *nucleus rotundus* (Fig. VIII, IX) of the diencephalon. It proceeds to the lateral part of the hyperstriatum forming a compact fibre tract in its passage through the mesostriatum (fig. VII—X).

The fibres help to constitute the *lamina medullaris externa* before entering the hyperstriatum. The striated appearance of the hyperstriatum is in great measure due to its infiltration by fibres of this tract. It is probable that a neurobiotactic principle is here exemplified. The presence of this afferent tract from the nucleus rotundus of the thalamus would tend to determine the origin of the lateral part of the hyperstriatum as an infolding of the pallium into which the tract originally poured the impulses carried by it.

Commissural fibres accompany the *tractus thalamo-frontalis externus* constituting the *commissura supra-optica dorsalis.* Though they are not heavily myelinated, the decussation of these fibres is clearly to be seen (Fig. VI). On each side the tract proceeds dorsally and caudally to merge with the external thalamo-frontal (Fig. VII, VIII).

2. *Tractus thalamo-frontalis medius.* This is a second afferent thalamo-striate tract situated medial to the external thalamo-frontal tract (Fig. IX). It arises from the *nucleus dorsalis* of the thalamus (Fig. XI) which lies dorsal to the *sulcus limitans* of His. Passing frontally and laterally the fibres of this tract mingle with those of the external thalamo-frontal tract and proceed to the frontal and occipital region of the hyperstriatum.

3. *Tractus strio-thalamicus internus.* The internal strio-thalamic tract is the main efferent tract from the corpus striatum to the brain stem. It can be recognised in the mesostriatum (Fig. VII) from which it passes medially (Fig. VIII) to take up a position medial to the afferent tracts to the corpus striatum. Some of the fibres join the anterior

commissure and cross to the other side as the *tractus strio-thalamicus cruciatus internus* (Fig. IX). Here they join the homolateral fibres of the opposite side (*tractus strio-thalamicus rectus internus*). The destination of these fibres in birds, as shown by EDINGER and WALLENBERG (1899) is the ventral thalamus and mid-brain.

f. Hypophysis and Epiphysis.

The hypophysis extending ventrally contains a funnel shaped prolongation of the median ventricle (fig. 8).

The epiphysis though damaged is clearly recognisable in a series of the sections.

IV. *General summary.*

The brain of the kiwi, for that of a bird, is remarkable for the great development of its olfactory lobes. In contrast with this the visual connexions are much reduced. Externally the usual subdivision of the avian cerebral hemisphere into *pars medialis* and *pars lateralis* cannot be seen.

A study of sections shows that the olfactory bulbs and lobes present a typical bulbar formation. The *area praepiriformis*, the nuclei of the septum, and the Area 28 of ROSE are recognisable. In accordance with the great development of the smell apparatus the ventral forebrain bundle, the fronto-epistriatic tract, and the olfacto-habenular tracts are well developed. Accompanying the olfacto-habenular tract is the *commissura telencephali superior* which is usually not seen in birds.

As is usual in birds the corpus striatum forms the major part of the cerebral hemisphere. The natural subdivison of the striatum is clearly revealed in the kiwi. The archistriatum (secondary epistriatum) can be recognised by the fact that it receives the fronto-epistriatic tract and is connected to its fellow of the opposite side by the inter-epistriatic commissure which is a conspicuous constituent of the *commissura anterior*. The palaeostriatum consists of the basal nucleus (*palaeostriatum primitivum*) and mesostriatum (*palaeostriatum augmentatum*). The mesostriatum in separated from the hyperstriatum by the external medullary lamina which extends from the ventricle medially to the ventral surface of the hemisphere, the line of separation here being indicated by the *fissura ventralis*. Vessels (hyperstriatal artery) enter this groove and accompany the external medullary lamina. These vessels are homologised with the lateral striate artery of reptiles (ELLIOT SMITH) and the claustral artery of man (SHELLSHEAR). The

fact that it deeply penetrates the hemisphere to supply the lateral part of the hyperstriatum indicates that this structure is pallial in origin, as this vessel represents a greatly hypertrophied pallial vessel. The basal nucleus and mesostriatum are supplied by palaeostriatal arteries indicating that together these masses form the palaeostriatum. This is in accordance with the ontogenetic studies on the bird's brain of Ariëns Kappers.

In this way the subdivision of the bird's brain may be linked up with those of the reptile and so, from work already published, homologised with the constituents of the corpus striatum of mammals. The palaeostriatum of birds, represented by the *palaeostriatum primitivum* and the *palaeostriatum augmentatum*, is homologous with the globus pallidus of mammals. The hyperstriatum corresponds to the putamen and caudate nucleus (neostriatum of Ariëns Kappers). Though the hyperstriatum in most birds is divided by the *lamina medullaris hyperstriati* into the *hyperstriatum superius* and the *hyperstriatum inferius* the sections under review do not exhibit this subdivision. Ariëns Kappers believes that the *hyperstriatum inferius* corresponds with the putamen and caudate nucleus of mammals and that a possibility exists that the *hyperstriatum superius* represents the claustrum which is also hypopallial in origin. The archistriatum forms the hypopallial part of the *nucleus amygdalae*.

The forebrain acts upon the brain-stem by the ventral forebrain bundle, and upon the ganglion habenulae by the olfacto-habenular tract. The strio-mesencephalic, occipito-mesencephalic, and septo-mesencephalic tracts connect it with the mesencephalon. The corpus striatum receives the external and medial thalamofrontal tracts from the *nucleus rotundus* and *nucleus dorsalis* of the thalamus respectively. Accompanying the external thalamo-frontal tract is the dorsal supra-optic commissure. The efferent mechanism of the corpus striatum consists of the direct and crossed internal strio-thalamic tracts which terminate in the ventral thalamus and mid-brain.

LETTERING USED IN THE FIGURES.

A. prp.	Area praepiriformis.
B. olf.	Bulbus olfactorius.
C. t. s.	Commissura telencephali superior.
Cap. ex.	Capsula externa.
Cap. h.	Caput hyperstriati.
Cblm.	Cerebellum.
Cer. H.	Cerebral hemisphere.
Co.	Cortex.

Com. anter.	Commissura anterior.
Com. interep.	Commissura interepistriatica.
Com. p.	Commissura posterior.
Ec.	Ectostriatum.
G. h.	Ganglion habenulae.
G. isth.	Ganglion isthmi.
Gl.	Glomeruli.
Hyp.	Hyperstriatum.
Hyp. a.	Hyperstriatal artery.
Lam. med. ex.	Lamina medullaris externa.
Lob. olf.	Lobus olfactorius.
M. C.	Mitral cells.
Mes.	Mesostriatum.
N. oc.	Nervus oculomotorius.
N. tr.	Nervus trochlearis.
N. trig.	Nervus trigeminus.
Nuc. d.	Nucleus dorsalis.
Nuc. l. l.	Nucleus lemnisci lateralis.
Nuc. prof.	Nucleus mesencephali profundus.
Nuc. m. l.	Nucleus mesencephali lateralis.
Nuc. r.	Nucleus ruber.
Nuc. rot.	Nucleus rotundus.
Nuc. sp.	Nucleus spiriformis.
Pal. a.	Palaeostriatal artery.
S. l.	Sulcus limitans.
T. op.	Tectum opticum.
Tr. fr. ep.	Tractus fronto-epistriaticus.
Tr. fr. ep. (p. d.).	Tractus fronto-epistriaticus (pars dorsalis).
Tr. oc. mes.	Tractus occipito-mesencephalicus.
Tr. op.	Tractus opticus.
Tr. olf.	Tractus olfactorius.
Tr. olf. hab.	Tractus olfacto-habenularis.
Tr. sep. mes.	Tractus septo-mesencephalicus.
Tr. st. mes.	Tractus strio-mesencephalicus.
Tr. st. th. int.	Tractus strio-thalamicus internus.
Tr. st. th. cruc. int.	Tractus strio-thalamicus cruciatus internus.
Tr. st. th. rect. int.	Tractus strio-thalamicus rectus internus.
Tr. th. fr. ex.	Tractus thalamo-frontalis externus.
Tr. th. fr. med.	Tractus thalamo-frontalis medius.
V.	Ventricle.
V. f. b.	Ventral forebrain bundle.

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Histology. — "*The histopathology of Lyssa in respect to the propagation of the lyssavirus*". By Mistress E. WINKLER-JUNIUS and J. A. LATUMETEN. (From the Psychiatric Neurologic Clinic at Utrecht). (Communicated by Prof. C. WINKLER).

(Communicated at the meeting of December 29, 1923).

Thanks to the kindness of Professor DE BLIECK and Dr. WINCKEL of the Veterenary University, we were in the opportunity of examining the nervous system of some dogs and rabbits inoculated with lyssavirus. The inoculationtime of the different cases diverged from seven weeks to three months. The animals were killed and submitted to an autopsy as soon as the first symptoms of the illness appeared. The questions, which after histopathological examination of the first case came to the front, diverged too much for our limited material to answer them all. We restrained our investigation therefore to one single question, a question that was given us by the clinical and experimental facts concerning lyssa. The clinical point of view, that the unknown virus of lyssa reaches the central nervous system by the peripherical nerves is often defended by the fact, that the duration of incubation is in direct proportion to the distance of the entrance spot from the spinal cord or medulla oblongata.

Experimental researches have established this point of view and proved that, the segment of the central nervous system corresponding to the inoculated limb, first becomes virulent, whilst from that segment the virulence spreads proximally and distally through the nervous system (SCHÄFFER).

According to DI VESTEA and ZAGARI, the lyssavirus does not propagate along the sheats of the nerve, but chooses the nerve-substance itself as a medium for its growth, viz. after inoculation with lyssavirus in the nervus ischiadicus the propagation of the virus is stopped, if directly after inoculation a more central part of this nerve is sectioned and cauterized.

However there remains a divergence of opinion on these points, in detail discussed in the *Handbuch* VON KOLLE und WASSERMANN by Professor Jos. KOCH. This author himself holds the opinion that

the lyssavirus reaches the central nervous system along the nerves as well as along the blood- or lymphvessels.

For the spreading of the virus along the nerves pleads:

1st the experiments of DI VESTEA and ZAGARI

2nd the experiments of SCHÄFFER

3rd that the blood has never been virulent

[„Fast alle Forscher sind der Meinung dasz das Virus im Blute nicht vorhanden ist" KOCH, Band VIII, KOLLE und WASSERMANN, pag. 835].

4th that subdural inoculation of blood from animals affected with lyssa never causes the lyssadisease.

Against the propagation of the lyssavirus along the nerves pleads:

1st the experiments of ROUX and MARIE by which is proved that only intradural and intracerebral inoculation of lyssavirus gives 100 % positive results, whilst endoneural injection remains uncertain.

The fact that the saliva of animals inoculated with lyssavirus is most times virulent, would also plead against the propagation exclusively along the nerves, if there were not the experiments of BERTORELLI. This author proved that one sided section of the nerves innervating the salivary glands, shortly before an subdural injection with lyssavirus prevents the infection of the saliva at the operated side. Moreover the fact that the saliva becomes virulent in the latter part of the incubation, when the central nervous system is already infected, makes it possible that from the medulla oblongata along Nervus facialis and Nervus trigeminus the infection of the salivary glands takes place.

We especially intended to see how far the histopathology of lyssa agreed with the clinical and experimental facts mentioned above. So:

first: *„whether peripheric nerves have altered and whether these alterations may prove in favour of a propagation along the nerve.*

, secondly: *how far a similar propagation of the lyssavirus persists in the central nervous system and*

thirdly: *whether the nervous path to the salivary glands has altered in such a way that these nerves may be considered as the medium through which the virus reached the glands.*

To answer these questions and to make the histopathological examination at the same time as complete as possible we subdued our material to different fixation and staining in a way as gives the annexed scheme. (See Table following page).

As soon as in the cornu *Ammonis Negri*bodies were found and besides the clinical fact there was also a histopathological proof,

Fixation.	Staining and purpose of the method.	Part of the nervous systeem.
I. Alcohol 96 %.	Toluidin cellstaining NEGRIbodies (LENZ)	Cornu Ammonis, med. spinalis, cerebellum, brainstem, cortex cerebri
II. Sublimate trichloretic acid.	Neuroglia staining FIEANDT.	Cornu Ammonis, tela, gyrus centralis
III. Alcohol 96 % + ammonia.	Neurofibrils RAMON Y CAJAL	med. spin, cortex, stem
IV. Formaline 10 %.	BIELSCHOWSKY neurofibril method. CAJAL's neuroglia method, fat staining	cortex, med. spinalis, cerebellum
V. FLEMMING's fluid.	ALZHEIMER's method for the myelin sheaths	cortex, med. sp., med. oblongata, peripheric nerves, brainstem, cerebellum
VI. ORTH-MÜLLER.	DOINIKOW's nervesheaths method	med. spinalis, peripheric nerves.
VII Cobaltnitrate.	DA FANO method for GOLGI apparatus	Cortex cerebri

that the animal had succumbed by lyssa, the nerves innervating the inoculated limb were examined on alteration.

Fig. I shows that the nerve everywhere has lipoid lumps. Although

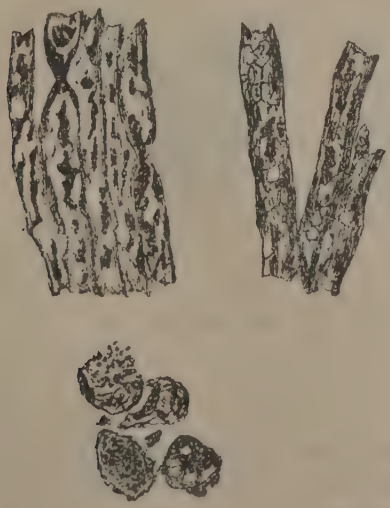

Fig. I.
Degeneration of the lumbal nerve at the inoculated side.

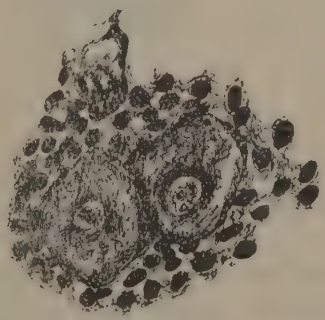

Fig. II.
BABES' Knötchen in the spinal ganglion of the inoculated side.

the degeneration of the nerve was not equally divided through the whole breadth of the nerve, there was sufficient proof that the nerve had been submitted directly or indirectly to a noxious influence. Therefore it was necessary to pursue the examination up to the spinal ganglion. It then appeared that the whole proximal part of the nerve was degenerated and the nervefibres in the spinalganglion as well were swollen and rich in lipoid lumps.

The nervecells in the ganglion had altered and were surrounded by phagocytes. Here and there „Babes Knötchen" (an accumulation of small cells in the midst of which were lying some skeletons of nervecells) were visible. (Fig. II).

Pursuing the radices up to the medulla, we got nearly the same image of a degenerated nerve as was shown to us by the more peripherical parts of the nerve. The swollen cylindre axis is lying in a hollow tube from which the myelin has totally disappeared (Fig. III). This degeneration of the roots could be followed into the white matter of the medulla spinalis, although between the degenerated tubes there were always fibres quite normally built. The nervecells in the segment of the medulla corresponding with the examined nerve had partly lost their *Nissl* lumps, partly were they recognizable only by a pale nucleus, surrounded by a mass of small gliacells (Fig. IV) *Negri*bodies were not found here.

Fig. III.
Lumbal root of the inoculated side with degeneration and loss of the myelin sheaths.

Fig. IV.
Neuronophagy.

Microscopical slices impregnated with silver showed that the intracellular fibrils either had disappeared or had clotted together to big threads (*Golgi* alterations fig. V).

Just as it was found in the spinal ganglion the degenerated cells lay scattered among quite normal cells, so that the examination

with a slight magnification at first gave the impression that the cellgroups had hardly suffered any loss.

Fig. V.
GOLGI alteration in a nervecell of the medulla spinalis.

Transversal sections of the adjacent proximal part of the medulla showed the following histopathological changes. All the nervefibres of the lateral columns of the inoculated side are destroyed, the anterior columns as well as the posterior columns are partly destroyed, which destruction is continuous with the group of degenerated fibres of the healthy side.

Pursuing the medulla spinalis proximally, it appears that mainly both the lateral columns have a fatty degeneration of their myelin but the posterior and anterior columns too have some destroyed fibres.

To value the alternations of the bloodvessels in the peripheric lumbar nerves and in the medulla spinalis was extremely difficult. Prepared as we were by the description in literature to find large infiltrations round the vessels and in the tissue, to find hyalin lumps in the walls of the bloodvessels, we were disappointed, when searching for these changes.

The bloodvessels in our slices showed a growing of their endothel and were studded with blood corpuscles, we found little haemorrhages in the peripheric nerve and somewhere round the vessels. The blood corpuscles however showed no trace of fatty degeneration, the haemorrhages proved to be of very recent date. So, although the alterations of the bloodvessels were in accordance with the result of other authors, as far as the degree of these alterations was concerned, it was impossible for us to place them above the changes of the nervecells and nervefibres. The serious degeneration of the myelin sheaths, the loss of the intracellular fibrils in the cells made it very probable that these changes had preceded the very recent infiltrations round the vessels.

Toluidin preparations of the lower part of the medulla oblongata.

where the nuclei Nervi XII, Nervi XI and Nervi X are to be found, offered a similar aspect of the state of their cells as is given by us for the cells of the medulla spinalis. In this part of the nervous system *Negri*bodies in the nervecells were found. Plate I, Fig. 1.

The more proximal part of the medulla oblongata, the brainstem, the cerebellum and some parts of the braincortex were submitted to different methods.

In all those parts we found degenerated myelin sheaths and cylindreaxes. The degeneration often consists in a tumefaction of the myelinsheath and a loss of myelin, whilst the cylindreaxis is preserved lying in the middle of the hollow tube. Fig. VI. No growing of neurogliacells along the destroyed sheaths was to be seen. Changes of nervecells of those parts were of different degree. The nuclei often had the most intensive changes. They partly had lost their membrane, were pale and swollen with only a single nucleolus stained red with eosine. Some times the nucleus was diffuse stained, and scarcely any structure was to be seen.

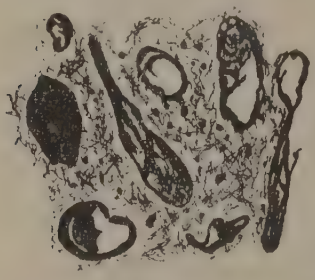

Fig. VI.
Degeneration of myelin sheaths.

Silverpreparations showed a granulation of the intracellularfibrils, some times alterations as described by GOLGI, the clotting together of the fibrils were present. The impregnation of the extracellular fibrils was some times very coarse just as it is to be found in ALZHEIMER's disease. This argentophily of the tissue round the fibrils is perhaps to be explained by the presence of a large quantity of demolition products, reducing the silver nitrate.

The *Purkinje* cells in the cerebellum had lost their *Nissl* lumps and their intracellular fibrils, and were often only recognizable by a partly destroyed nucleus in the neighbourhood of which *Negri*-bodies were lying. Plate I, fig. 2.

The cells of the cornu *Ammonis* were the principal seat of the *Negri* bodies, but our preparations gave no proof of these bodies, being generally found in relatively healthy parts. In cells which had scarcely undergone any change, as well as in cells totally destroyed, these *Negri*bodies were present. For instance there was a lack of *Negri*bodies in the cells of the medulla spinalis; in the brainstem and in the medulla oblongata, however we found a large number of these bodies although the medulla spinalis as well as the brainstem and medulla oblongata were the seat of serious alterations of the nervecells and nervefibres

Benedek and Posche in their monography as well as Francesco di Felice in his paper dispute the parasitic nature of the *Negri*bodies and both these authors are convinced that fragments of the nucleus of the nervecell take part in the formation of these *Negri*bodies.

Touching our material in this question we could state that often *Negri*bodies were lying next to a quite normally built nucleus, on the other hand we found through the whole nervous system changes of the nuclei, that pleaded for the opinion of both authors. So we found coarsely granulated nuclei, some of which granules entering into the cytoplasma (Fig. VII). The *Lenz*method, for the staining of *Negri*bodies too demonstrated big granules in the nucleus assuming the same stain as the *Negri*bodies do, whereas in normal brains we did not find such granules. Sometimes the whole nucleus had assumed a diffuse red stain (with Lenz method) as if the different granules were dissolved.

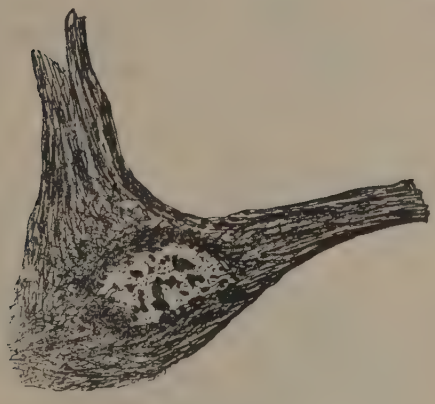

Fig. VII.
Granules from the nucleus entering the cytoplasm.

It was not our intention to enter deeply into the question concerning the nature of the *Negri*bodies; curiosity however stimulated us, when we treated our slices of the cornu *Ammonis*, rich in *Negri*bodies, with nuclease, in order to see whether these corpuscles as well as the nuclei could be destroyed.

Dr. van Herwerden was so kind as to teach us how to arrange these experiments. The result was as shows Plate I, fig. III that only skeletons of the nucleus were visible after the experiment, but that the *Negri*bodies had obstinately resisted the resolving power of the nuclease. They had kept the same form and stain as the *Negri*bodies which were treated with boiled nuclease that had lost its destructive power. Plate I, Fig. IV.

But these negative results do not exclude that the *Negri*bodies are built from material of the nucleus, especially by oxyphile elements, which are not destroyed by the nuclease. Our experiments only pleaded for the fact that the chromatine elements of the nuclei probably do not take part in the formation of these bodies.

As we had at our disposal section as well as fixation of our material, we succeeded in the impregnating of the Golgi apparatus

of the nervecells and as changes of this apparatus in pathological cases are seldom described, we think it justified to demonstrate them in this essay, although the significance of this apparatus and its changes is not at all known and has apparently nothing to do with the questions we treat of in this essay.

WILDER PENFIELD in his paper "Alterations of the GOLGI apparatus in nervecells" writes: "The GOLGI apparatus presents normally in the great majority of cases a complete attenuated reticulum with many varicosities or lacunae... The structure is confined to the cytoplasma never encroaching on the nucleus or the periphery of the cells.... The whole structure appears rarely in one half of the cytoplasma only. It may be hypertrophied or meagre but under normal conditions the general pattern is *surprisingly constant* [1]) for each type of cells Fig. VIII shows the apparatus in nervecells of normal brains.

According to CAJAL the reticulum should be more resistant to pathological agents than are the neurofibrils.

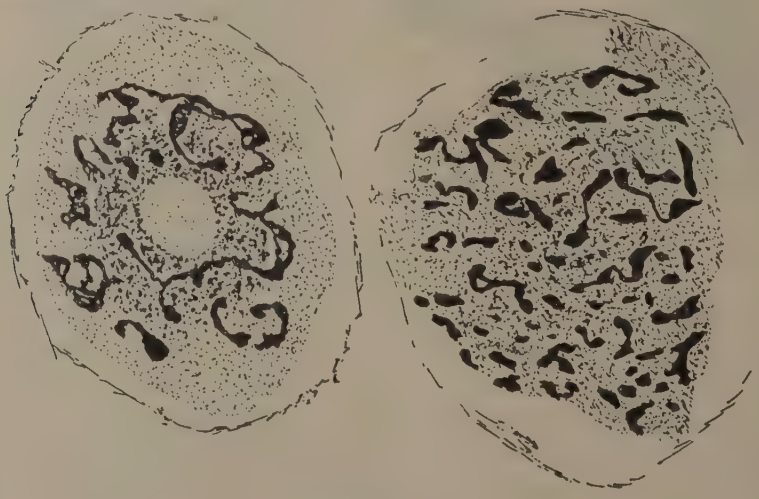

Fig. VIII.
GOLGI-apparatus in normal cells.

PENFIELD divides the reaction of the apparatus in the nervecells after sectioning the nerves into three stages.

1st Displacement of the unbroken apparatus to the periphery of the cell and away from the axonehillock *retispersion.*

2nd Dissolution of the reticulum *retisolution.*

3rd *Reconstruction.*

[1]) The italics are ours.

In the cases of lyssa the most conspicuous change of the GOLGI-apparatus was the retispersion as shows fig. IX. Round the nucleus

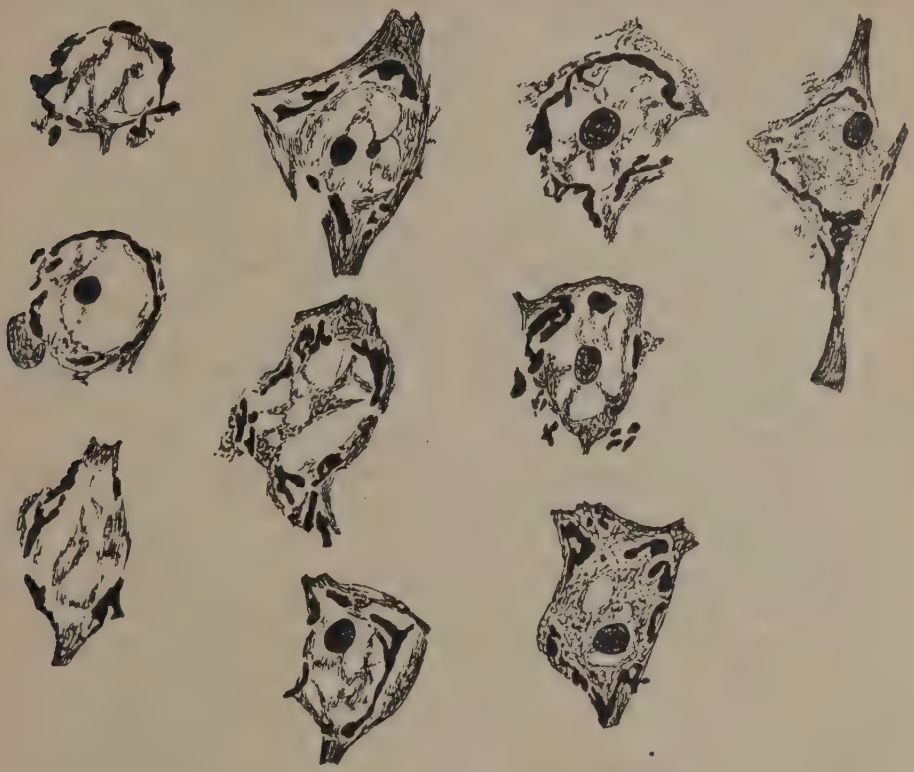

Fig. IX.
Retispersion and *stretching* of the GOLGI apparatus.

the apparatus has disappeared. The second constant change was the *stretching* of the apparatus. Instead of the small curvations as seen in normal cells, the meshes of the changed reticulum are bordered by stretched threads. Perhaps this stretching is due to the retispersion viz. to the fact, that the apparatus has to occupy a larger sphere round the nucleus than in normal cells.

Changes of the neuroglia were only of a slight degree.

Round the changed or destroyed nervecells the number of neurogliacells had augmented, amöboid gliacells were scarcely found along the degenerated nervefibres. There was a total lack of neurogliafibres. Resuming the changes of the different elements described above, we get in the first place.

1st *Changes of the nerve fibres.*

Serious degeneration of nervesheaths, swollen cylindre-axes in the

peripheric lumbar nerve of the inoculated side, in the lateral columns of the medulla spinalis and everywhere scattered among normal fibres in parts of the cerebellum brainstem and cerebrum.

2nd *Changes of nervecells.*

a. Destruction more or less of the nucleus

b. Dissolution of *Nissl*lumps

c. Granulating of nervefibrils

d. GOLGI alterations of the intracellularfibrils

e. Presence of *Negri*bodies

f. Retispersion and stretching of the GOLGI apparatus

g. BABES' Knötchen mainly in the ganglion spinalis

3rd *Changes of the bloodvessels.*

a. Vessels studded with cellular elements

b. Small infiltrations round the vessels of haematogenous elements

c. Growing here and there of the endothelcells

4th *Changes of the neuroglia.* Very slight.

Trying to answer our first and second question with these facts it is obvious, that of all the changes those of the nervefibres, especially of the myelinsheaths are the most conspicuous.

As to the degeneration of the peripheric nerve it is certainly not a WALLER degeneration for:

1st. the cylindre-axis are much less destroyed than are their myelinsheaths.

2nd. there is scarcely any reaction from the side of the cells of SCHWANN.

As to the degeneration of the fibres in the medulla spinalis there is no question of a system degeneration. The destroyed myelinsheaths and swollen cylindre-axes can not be pursued up to their nervecells, viz. in the neighbourhood of the different cellgroups, in the medulla scarcely any destruction of fibres is to be found. So the destroyed myelin can be explained only by a direct influence of the virus.

Admitting that the lyssavirus propagates along the lymph or bloodvessels and the virus itself or its toxines entering the nerve on different spots destroys the myelin, than the question arises why precisely the nerve of the inoculated muscles has degenerated in its whole length without any interruption whilst other peripheric nerves either have a degeneration of their roots and the most central part of the postganglionic part or have no degeneration at all.

Admitting therefore that the virus from the inoculated spot reaches the central nervous system, it does not in the least exclude that along the vessels as well the virus is transported. In the medulla

spinalis the destroyed fibres are mainly lying in the peripheric part of both the lateral columns, but they are not entirely lacking in the peripheric zones of the dorsal and ventral columns. As to the degeneration of the right (inoculated) side of the medulla, the supposition lies at hand that the virus as soon as it has reached the spinal chord by the anterior or posterior roots, takes the most peripheric lying myelinsheaths as a medium for its growth. But then it has to be explained that there is scarcely any difference between both the lateral columns and even in a transversal section of the lumbal medulla, it is difficult to see which side is the inoculated side.

However, this histopathological fact fully agrees with the experiments of Roux, which teach us, that the virus directly after its arrival in the medulla infects the opposite side, so that the peripheric nerve of this not inoculated side becomes virulent, before the proximal and distal parts of the medulla are infected. Supposing, the neurogliareticulum, in which meshes the nervefibres are lying, undertakes the transport of the virus, then it is not explained why the more centrally lying myelinsheaths have not altered. Our opinion in this question is, that probably the liquor cerebrospinalis surrounding the medulla as well as the anterior and posterior roots undertakes also the transport. (The experiments of Daniel Konradi recently proved the virulence of the liquor cerebro-spinalis, a fact that hitherto was denied in literature).

The answer to the third question, whether the virus reaches the salivary gland by growing along the afferent or efferent nerves, required a complete examination of a larger part of the brainstem of Nervus facialis with chorda tympani, and of the ganglion Gasseri with Nervus trigeminus, especially its ramus lingualis. Annexed scheme demonstrating the innervation of the salivary gland of rabbits and dogs shows what nerves were submitted to examination.

Scheme of the examinated parts of N. V. and V. VII.

The brainstem, fixated in Flemmings fluid was sectioned in a series of transversal slices in order to stain them with Fuchsin Lichtgrün.

Fig. X representing a section through the brainstem and the roots of N. V. shows the degenerated fibres especially in the portio minor, though also the portio major has swollen or destroyed fibres.

The nervecells in this section and through the whole brainstem have more or less changed and many *Negri*bodies are found here. Pursuing the roots up to the ganglion *Gasseri* partly the fibres have degenerated though by far not the greater part. Longitudinal sections through the third branch of this ganglion consisting of N mandibularis with N lingualis show a serious degeneration especially of the N. lingualis. Also the cells of the ganglion oticum and the fibres

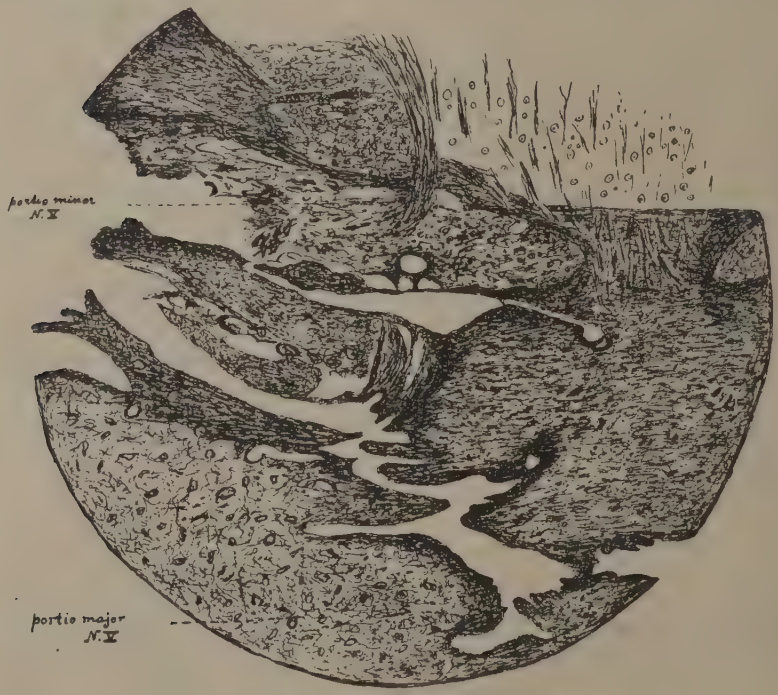

Fig. X.

Section through the roots of *N.V.* Degeneration of the portio minor- and portio major *N.V.*

passing that ganglion show a serious degeneration. Fig. XI. Pursuing the N lingualis up to the salivary gland we saw this nerve degenerated along its whole length (Fig. XII). So there is no doubt that the nervous path connecting the central nervous system with the salivary gland is more or less destroyed.

A section through the brainstem and the roots of N facialis shows a degeneration of the facialis roots, though of a slight degree and more or less of the vestibularis roots. Also the part of the corpus trapezoides lying between both roots is partly destroyed, so that it seems that especially the lateral peripheric part of the brainstem

has been influenced by the virus (Fig. XIII). A more distal section of N facialis and its branch the chorda tympani proves, that both these nerves have degenerated, so that the second nervous path too, connecting the brainstem with the salivary gland prove to be degenerated. As to the cellular elements connected with both these pathes we found in the ganglion *Gasseri* as well as in the ganglion oticum serious alterations of nervecells. We did not succeed in getting sufficient slices of the ganglion geniculi.

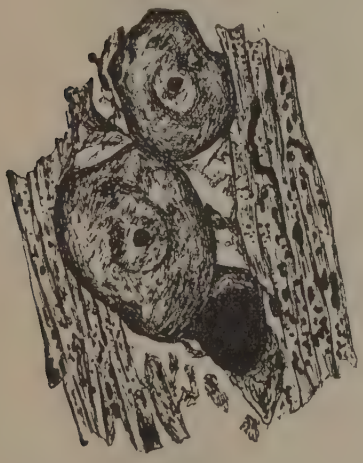

Fig. XI.
Degenerated lingualis fibres passing the ganglion oticum.

Fig. XII.
Nervus lingualis entering the glandula sublingualis.

On the other hand the N abducens, the more distal part of the N. mandibularis, N. ophtalmicus, and N. maxillaris proved to contain for the greater part normal fibres as well as nerves more proximally entering or leaving the brain e.g. the N. opticus and N. oculomotorius had no changes at all.

Resuming we found the brainstem seriously changed, the alterations of the nervecells with their *Negri*bodies, the destroyed myelinsheaths especially in the peripheric lateral parts of the stem, proved that this part of the central nervous system had been strongly influenced by the noxious virus.

Pursuing the different roots leaving the brainstem we found both roots of the N. trigeminus degenerated especially the portio minor. The facialis roots also showed some degenerated fibres, but in a far less degree than the roots of the N. trigeminus. Of the branches of the ganglion *Gasseri*, the N. lingualis was the most destroyed nerve.

This result was of some importance in connection with the question, whether the lyssavirus chooses the nervous path to reach the salivary gland.

Suppose we did not find any degeneration in the nerves, innervating the salivary gland in casu the N. lingualis with the fibres

Fig. XIII.
Degeneration of the roots of N vestibularis and N facialis.

of the chorda tympani, it would be evident that the virus had not reached the gland along the nerves, because the histopathology of lyssa gave sufficient proof that the virus has a noxious influence especially on the nervefibres and its myelinsheaths.

However as the nervefibres, connecting the nervous system with the salivary gland have indeed changed and these changes seem to be of older date than the changes of the side of the bloodvessels, it is most probable that these changes of the fibres are directly brought about by the lyssavirus.

Therefore the histopathological changes of the brainstem and the

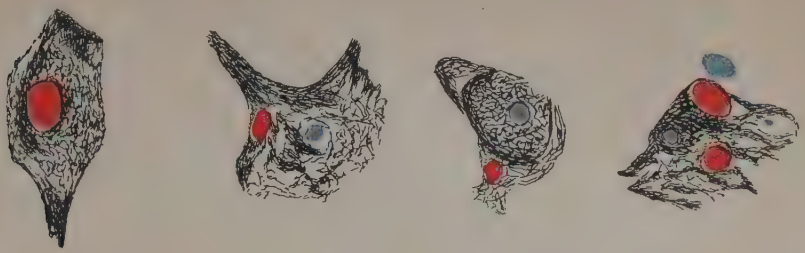

Fig I

Fig II

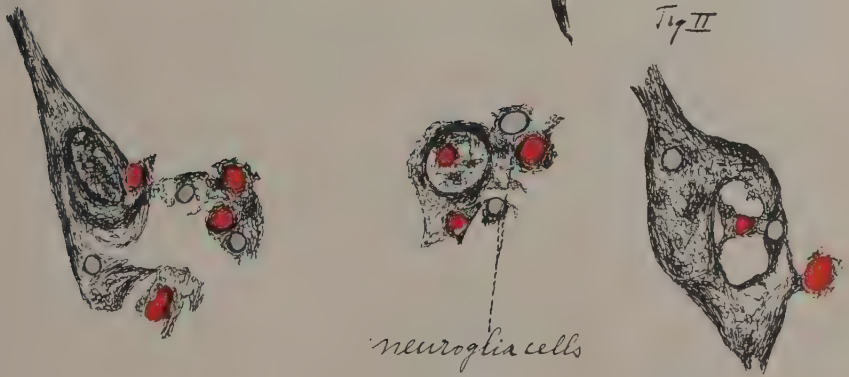

Fig III

Fig IV

nerves innervating the salivary gland make it most probable that the lyssavirus reaches the salivary gland along the nervous path.

The histopathology of lyssa fully agrees with the experimental results teaching 1*st that only in the second part of the incubation the saliva becomes virulent.* 2*nd that the virulence of the central nervous system precedes that of the salivary gland.* (experiments of *Bertorelli*).

As to the other side of the question whether the nervous path is chosen *exclusively* by the virus, we think the most detailed histopathology of the lyssa brain incompetent to solve this problem.

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DESCRIPTION OF PLATES.

Plate I.

Fig. I. Negri bodies in the nervecells of the medulla oblongata. Lenz staining.

Fig. II. Negri bodies in the Purkinje cells.

Fig. III. Negri bodies having resisted the influence of nuclease; Neurogliacells having lost the greater part of their nuclei surrounding the nervecells.

Fig. IV. Negri bodies in slices treated with boiled nuclease.

Physics. — "*Magnetic Researches.* XXVI. *Measurements of Magnetic Permeabilities of Chromium Chloride and Gadolinium Sulphate at the Boiling Point of Liquid Hydrogen in Alternating Fields of Frequency 369,000 per Second.*" By G. BREIT, National Research Fellow, U.S.A., and H. KAMERLINGH ONNES. (Communication N°. 168c from the Physical Laboratory at Leiden.)

(Communicated at the meeting of December 29, 1923).

§ 1. *Introduction.* It has been suggested by EHRENFEST [1]) that at very low temperatures paramagnetic substances may show phenomena of hysteresis. The experiments reported on in this communication were made in order to see whether this effect is present at reasonably high frequencies. The quantities measured were magnetic susceptibilities. The measurements were made on samples previously used in steady field determinations to as to enable a direct comparison. The measurements made do not give one sufficient confidence to claim great numerical accuracy of the results. However, they seem to indicate definitely that the order of magnitude of the susceptibilities for steady and alternating fields is the same. The numerical values obtained for both salts are smaller than the values obtained in direct fields and the apparent consistency of trial measurement given below suggests that this discrepancy may be not due to experimental error.

§ 2. *Methods and Apparatus.* The method was similar to that described by BELTZ [2]). Two electron tube circuits (Nr. 1, Nr. 2) were set up to generate sustained oscillations of high frequency. The frequencies of the two were adjusted so as to be nearly integral multiples of each other. A two stage audio frequency amplifier was coupled loosely to both. The audible beats produced in the amplifier were made to give beats with an audible note produced by a third electron tube circuit, say Nr. 3. The paramagnetic sample was put into the inductance of circuit Nr. 1. The cryogenic apparatus surrounding the sample was placed inside the same coil. The coil was shielded on its inside by means of tinfoil. The tinfoil was cut into 8 segments so as to allow the magnetic field to pass to the inside of the shield. The cryogenic

[1]) P. EHRENFEST, these Proc. 23, p. 989; Leiden Comm. Suppl. N°. 44*b*.

[2]) Phil. Mag. 44 (1922) p. 479.

apparatus consisted of two non-silvered vacuum glasses — the outside being used for air and the inside for hydrogen in the usual manner. Fig. 1 shows the shielded box B into which circuit Nr. 1 was put. C is the top of the inductance coil shield, L is the lid of the box, P is the packing. Under the hood circuits Nr. 2 and Nr. 3

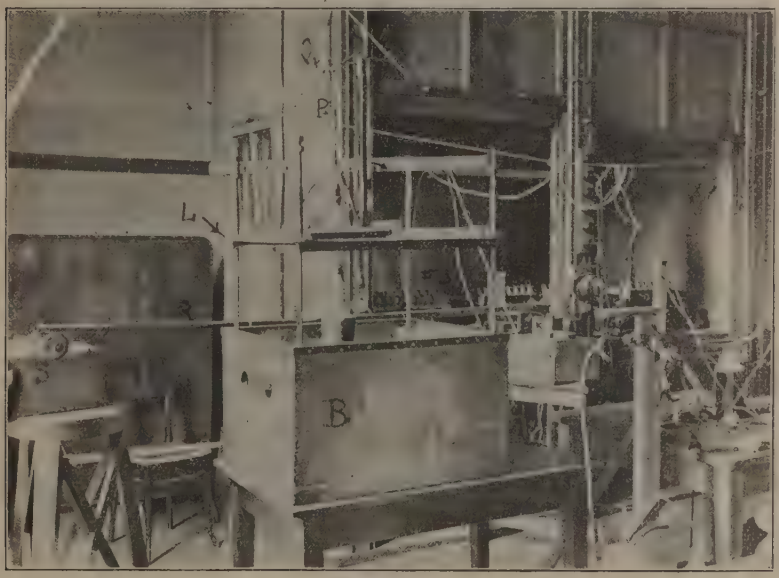

Fig. 1.

are arranged as shown. The amplifier A is on the table to the right. The small condenser K (used at a place where it has 313 $\mu\mu f$ capacity) is connected in series with a larger fixed condenser (capacity 2847 $\mu\mu f$) between the filament and grid of circuit Nr. 2. The rod R 270 cms. long controls the motion of K, being attached to a screw adjustment S. Adjusting S changes the frequency of circuit Nr. 2 by small amounts. On fig. 2 a view is given of the inside of the box B, the shield of the coil C and its windings W being plainly visible. The shield segments are connected by the wire D and when the lid is lowered the wire D is connected to the box shield by a short wire and a clip. The windings of the coil W are supported by a piece of glass tubing to which they are well fastened with paraffine. The upper part of the shield is made of pasteboard tubing thoroughly boiled in paraffine and hardened by several coats of shellac. The filament and plate batteries are also shown on this picture. The filament rheostat which is shown on the right wall has been short circuited in the final measurements so as to eliminate fluctuations in the

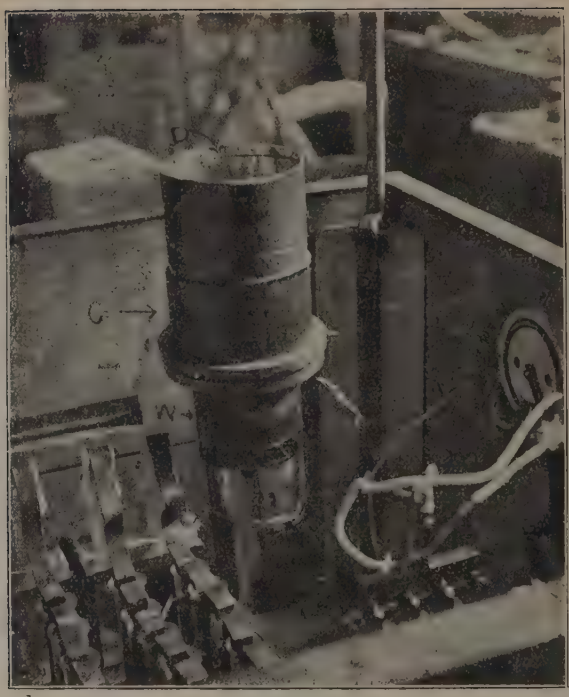

Fig. 2.

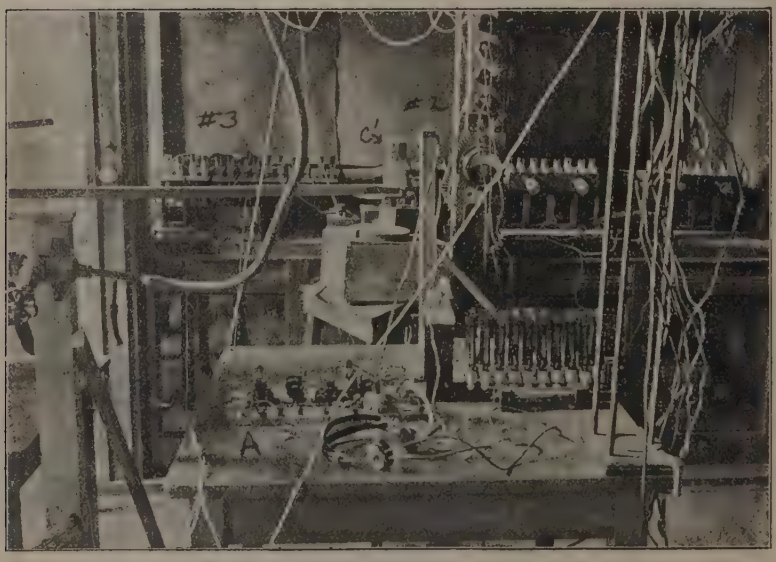

Fig. 3.

filament current. Fig. 3 shows circuits Nr. 2, Nr. 3 and the amplifier in more detail. C' is the inductance of Nr. 2. The valves used have been MULLARD or PHILIPS receiving valves. The circuit connections in Nr. 1 and Nr. 2 have been those of the usual HARTLEY circuit a tap off at the middle of the coil leading to the filament and the extremities going to the grid and plate battery terminals. A condenser was connected across the coil terminals. All the parts of circuit Nr. 1 were thoroughly shielded. The coupling to the amplifier A was accomplished by a wire entering the box B through a hole on the side opposite to that shown Fig. 1. This wire was connected to the input of A either directly or through a transformer. The input terminal of A was also coupled by a wire to circuit Nr. 2. The loose end of this wire was stretched in over a rope towards circuit Nr. 2. Circuit Nr. 3 was controlled by a pulley arrangement allowing one to turn its condenser plates without introducing capacity due to the observer's body. The observer was situated at the outstanding corner of B in Fig. 1. The telephones were used on the observer's head. The coupling in this apparatus was sufficiently loose to make the capacity effect through the phones negligible for the small motions of the observer's body during the measurement. The liquid air in the vacuum glass inside C produced sufficient cooling of it to cause unsteadiness. It thus was necessary to have a rapid method of measurement.

For this purpose a system Q (Fig. 1) was attached to the rod sliding through P. Q consists of a horizontal metal rod carrying three collars. The central one is attached to a vertical guide passing through a collar almost directly above P. The motion of this guide can be controlled by the lever shown. The collar on the right is connected to P by a vertical rod and the collar on the left carries a glass rod from the lower end of which is suspended a glass tube by means of a thread. The metal rod passing through P ended well inside the tube of German silver supporting P and was thus shielded from the action of the magnetic field of C. To the lower end of that rod a glass tube was fixed with sealing wax and this tube supported the paramagnetic specimen. The paramagnetic specimen and the suspended tube could be thus moved in and out of C simultaneously. Inside the glass tube a single or several copper wires were put and these were selected in such a way as to make the number of beats per second in the upper and lower positions the same. The combination was recorded and a subsequent calibration determined the magnitude of the effect. The range of motion was fixed by permanent stops on the sliding rods. This was necessary

because balance in the end positions was found not to mean always a balance in the intermediate ones.

§ 3. *Preliminary Tests.*

a. Necessity of shield for coil. The fact that the coil had to be shielded on the inside was ascertained by first trying the arrangement without the shield. A glass bottle lowered inside the coil by a thread produced an appreciable effect on the beats. This effect was absent when the inside of the coil was covered with thin tinfoil strips.

b. Bending of box lid. Many blank tests have been made to see whether the strains in the lid due to the up and down motion of the rod system Q affected the beats. No such effect was observed.

c. Interaction between circuits has been observed to be generally very small. Thus by adjusting the beat frequency between Nr. 1 and Nr. 2 to a low value like 100 no tendency of pulling into one has been noted. Putting a paramagnetic sample in and then out again under these circumstances did not alter the beat frequency noticeably. The beat frequency employed in the measurements was of the order of 500 or 1000 per second and the interaction must have been still smaller. A further test of this was made by first compensating a sample by changing K, say with set Nr. 1 going at a higher frequency than Nr. 2; then Nr. 2 was set to the higher frequency and the measurement repeated. The following table gives the results of the changes of the scale S.

	First Side.	Second Side.	Difference.	
Iron wire Nr. 7 in suspended tube	22.3	22.5	— 0.2	— 1%
Iron wire Nr. 8 in box . . .	13.9	13.2	0.7	5.4%
Combined action	34.3	33.6	0.7	2%

The iron wire Nr. 8 was here put in a tube fastened to the paramagnetic sample in the warm condition with thread and sealing wax. This tube was approximately at the axis of the coil while Nr. 7 was in the suspended tube almost at the shield. There appears to be no true systematic difference between measurements made with the two frequencies reversed.

d. Direct test of balance. It so happened that a copper wire (2 N) when used in the suspended tube balanced by its diamagnetic eddy current effect the paramagnetic effect of iron wire (Nr. 8). When the

effect of 2 N was balanced by means of K, 13.4 scale divisions were necessary while Nr. 8 took —14.2 scale divisions. The residual effect of the combined action was estimated at —0.5. The equation $14.2 = 13.4 + 0.5$ is satisfied to about 2 %.

e. Effect of length of paramagnetic sample. To test this four iron wires of different lengths were cut from one piece. They had lengths: 6.73 cm., 7.80 cm., 10.00 cm. Two glass tubes were fastened to the tube of $CrCl_3$ in the warm condition. Tests with these were made on May 12 and on July 13, 1922. In the first set of tests the paramagnetic action of the iron wires used in one of the tubes at the axis was balanced by the diamagnetic action of suitably chosen combinations of copper wires put into the suspended tube and besides a copper wire was put into the other tube at the axis so as to take up the bulk of the paramagnetic action of the iron wire. The measurements were then repeated with the iron wires turned upside down. No difference of using one end up rather than the other was found thus showing that the iron wires are homogeneous. Both sets of measurements agreed in showing that the effects of the wires having lengths 7.80 cm. and 8.80 cm. are nearly equal and slightly greater than the effects of wires having lengths 6.73 cm., 10.00 cm. The effect of the wire having 10 cm. length was only slightly smaller than the maximum. The copper and iron wires used at the axis were made to change places and the observations were confirmed the effect of the interchange being very small. On July 13 further confirmation of the observed effect of length was obtained the effect of the iron wires being this time compensated by changes in K. The number of scale divisions of S required for compensation was for the same wires in increasing order of length 18.4, 22,6, 22.5, 21.1.

The reason for the decrease in the paramagnetic effect at 10 cm. is not clear. The increase in the region of 7 cm.—8 cm. must be due to the simple increase in the length in a fairly homogeneous field because $\frac{22.6}{18.4} = 1.17$ while $\frac{7.80}{6.73} = 1.16$. It may be that the shield causes a peculiar distribution of magnetic field resulting in the upper portion of the 10 cm. wire being in a stronger field in the "up" position than in the "down".

f. Effect of criterion of compensation. When the beat frequency of circuits Nr. 1 and Nr. 2 becomes nearly equal to the frequency of Nr. 3, it becomes at times difficult to distinguish them. Also in theory one cannot deny a possible action of Nr. 3 on the combined system Nr. 1 and Nr. 2 and thus an effect on their beat frequency. If these effects

are present, one should expect the result of making settings by adjusting beats to zero and by adjusting beats to a fixed number to be different. The results however indicate that this effect is absent. Thus 13.2 and 13.1 are the readings obtained on the scale S by the two methods.

g. Calibration. Since all the changes in the frequency are very small the change in the frequency is very nearly proportional to the change in capacity or to the change in inductance that causes it. Therefore changes in K necessary to compensate for two different changes in inductance are proportional to these changes. If one change in inductance is known or if its meaning as a susceptibility is known the other is derived by multiplication into the ratio of the two settings of K. This is the principle of the calibration employed. The calibration divides itself into the following parts:

a. *To produce a change in inductance which has a direct interpretation as a susceptibility.*

For this purpose two glass tubes were attached to the paramagnetic sample at opposite sides of a diameter — while the sample was in the warm condition. Copper wires accurately drawn and measured could be inserted into these The length of the wires was nearly equal to the length of the column of paramagnetic substance employed. Roughly the wires may be said to exclude the high frequency magnetic field from their interior. To a first approximation they are therefore equivalent to a material of susceptibility $-\frac{1}{4\pi}$. If the positive effect of a paramagnetic sample is equal to the effect of a wire of a certain size, its susceptibility must be then equal to $\frac{1}{4\pi}$ times the ratio of the volume of the wire to the volume of the substance. Since the field is not quite excluded from the interior of the wire, its diamagnetic action is not quite as large as we have just supposed but a correction for this may be applied. Taking the field to be a homogeneous one along the axis of the wire the correction factor is $-\mu\beta(q)+1$ where

$$\beta(q)=\frac{2}{q}\,\frac{ber\,q\;bei'\,q-ber'\,q\;bei\,q}{ber^2\,q+bei^2\,q}$$

where $q=\sqrt{\frac{4\pi\mu\omega}{\sigma}}\,a$, *ber* and *bei* are the KELVIN functions, and μ, σ, a are respectively the permeability, resistivity, and radius of the wire used at the frequency $\frac{\omega}{2\pi}$.

b. Now the paramagnetic sample was balanced against wires put not in its immediate neighborhood but at the shield. A correction factor must be applied for this. The factor was determined experimentally by balancing the effect of the same wire by K, first at the tubes attached to the paramagnetic sample and second in the suspended tube used in the compensation of the paramagnetic salt. The ratio of the readings on S gave the correction factor. The determinations of the correction factor vary somewhat and a considerable part of the experimental uncertainty is due to this.

c. Finally a determination had to be made of the ratios of the changes in K which had to be made in order to compensate the change in inductance produced by the combination of wires which compensated the paramagnetic sample and the changes produced by the standard accurately drawn wires.

§ 4. *Results for anhydrous Chromium Chloride at the boiling point of hydrogen.*

At a frequency of 3.69×10^5 the sample of chromium chloride was balanced by a combination of wires the effect of which was soon afterwards compared with the effect of one of the standard wires ($2N$). The ratio of the effects of the combination of wires to the effect of $2N$ as measured by K was $\frac{6.0}{6.3}$. The correction factor due to the inhomogeneity of the field as measured on $2N$ was $\frac{6.3}{4.6}$. Finally the correction for the length (the sample of chromium chloride was larger than $2N$) was $\frac{18.3}{21.5}$. (This was determined from the results on iron wires cited above). The resultant correction is then $\frac{6.0}{6.3} \times \frac{6.3}{4.6} \times \frac{18.3}{21.5} = 1.11$.

Again at the frequency 3.69×10^5 the wire $2N$ has a $q = 9.01$ and hence $1 - \beta(q) = 0.841$. Thus the volume susceptibility of the wire is $\varkappa = -\frac{0.841}{4\pi} = -0.0669$. Now the radii of the wire and of the sample were 0.705 mm. and 3.5 mm. respectively. Hence the volume susceptibility of the sample is $\varkappa = 1.11 \times \left(\frac{0.705}{3.5}\right)^2 \times$ $\times\ 0.06692 = 0.00305$. The weight of chromium chloride was 3.192 grams and its length 9.7 cm.; the density is $\frac{3.192}{\pi \times 9.7 \times (0.35)^2} = 0.85$

and the specific susceptibility $\chi = 0.0036$. The value obtained in direct fields, according to unpublished results of Dr. H. R. WOLTJER is 0.0048 [1]).

§ 5. *Results for Gadolinium Sulphate at the boiling point of hydrogen.* At the same frequency of 3.69×10^6 the gadolinium sulphate was balanced against a different combination of wires which when compared with $2\,N$ had an effect smaller than $2\,N$ in the ratio $\frac{1.48}{10.1}$. The length of the sample was 8.74 cm. and the corresponding correction $\frac{18.3}{22.7}$. The weight of the sample is 2.897 grams. The specific susceptibility is hence $\chi = 0.00051$. The value obtained at the same temperature for the same sample in a steady field was 0.0010 [2]). Measurements on manganic and nickel sulphate have been also made and gave results of the same order of magnitude as those for steady fields.

§ 6. *Conclusions and Discussion.*

A. The order of magnitude of the susceptibility is unchanged if the frequency is increased to 3.69×10^6.

B. The results seem to indicate that the susceptibility is smaller than for direct fields. The values obtained in alternating fields for $CrCl_3$ and Gd sulphate are 0.75 [3]) and 0.51 [4]) respectively of what

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[1]) This value is obtained by the method of weighing a rod of the material in an inhomogeneous magnetic field (KAMERLINGH ONNES and PERRIER, these Proc. 16, p. 689, Leiden Comm. N°. 139*a*). However, the susceptibility seems to depend on the field strength, decreasing with increasing magnetic force. The value given relates to a field ranging from 4500 gauss at the top of the rod to 220 gauss at the bottom. The limit for very weak magnetic fields may be about 20% higher (as found by *extra*polating the susceptibility-magnetic force curves), so the ratio 0.75, given in § 6B for the susceptibilities in alternating and direct fields, may be too large.

[2]) KAMERLINGH ONNES and OOSTERHUIS, these Proc. 15, p. 322, Leiden Comm. N°. 129*b*, § 6. However, it has to be pointed out, that it was not sure the sample was really at the temperature of the bath, as it appeared afterwards, that in the experiments of KAMERLINGH ONNES and OOSTERHUIS it took some 4 hours before the susceptibility had taken a definite value, probably owing to lag in the temperature equilibrium. Even in order to avoid this difficulty and to ensure a better temperature equilibrium of the powder and the bath, the tubes for the magnetic investigations were later on not evacuated but filled with a small quantity of non condensing gases (hydrogen or helium). The value 0.00051 obtained with the present tube is probably too low.

[3]) See note § 4.

[4]) See note § 5.

they are in direct fields. However, it would be preposterous to conclude that the susceptibility is actually decreased by the amount found. Further work will be necessary for that. The choice of the place of the suspended tube was rather unfortunate. It was situated rather close to one of the slits in the tinfoil. Even though capacity effects appear to be absent this is dangerous because the magnetic field in the neighborhood of the slit is not homogeneous. It is possible that the divergence between the values for alternating and direct fields is due to insufficient caution in the manipulation of the suspended tube and a slight displacement of it during the experiment. This would hardly explain, however, the similarity [1]) of the results for the two substances investigated.

The writers wish to express their sincere thanks to Dr. H. R. WOLTJER for help in comparing the results with those in steady fields and for making unpublished results of his measurements available.

[1]) Especially, if the value 0.75 is too high and 0.51 too low (see notes §§ 4 and 5, this similarity is perhaps not only qualitative, but also more or less quantitative.

Mathematics. — *"On a non-symmetrical affine field theory."* By Prof. J. A. SCHOUTEN. (Communicated by Prof. H. A. LORENTZ.)

(Communicated at the meeting of October 27, 1923).

1. *Introduction.* In his last publications [1]) EINSTEIN has given a theory of gravitation which only depends on a symmetrical linear pseudo-parallel displacement ("affine Uebertragung") and a principle of variation. From the equations, that result in this case, we see that the electromagnetic field only depends on the curl of the electric current vector, so that the difficulty arises that the electromagnetic field cannot exist in a place with vanishing current density.

In the following pages will be shown that this difficulty disappears when the more general supposition is made that the original displacement is not necessarily symmetrical.

The equations which define such a displacement are

$$\nabla_\mu v^\nu = \frac{\partial v^\nu}{\partial x^\mu} + \Gamma'^{\nu}_{\lambda\mu} v^\lambda$$

$$\nabla_\mu w_\lambda = \frac{\partial w_\lambda}{\partial x^\mu} - \Gamma'^{\nu}_{\lambda\mu} w_\nu \ ,$$

in which the parameters $\Gamma'^{\nu}_{\lambda\mu}$ (with an accent to distinguish them from the $\Gamma^{\nu}_{\lambda\mu}$ of a symmetrical displacement) are not symmetrical in λ, μ.

EINSTEIN [2]) has defended the use of symmetrical parameters with the remark that in the non symmetrical case not only

$$\frac{\partial w_\lambda}{\partial x^\mu} - \Gamma'^{\nu}_{\lambda\mu} w_\nu$$

but also

$$\frac{\partial w^\lambda}{\partial x^\mu} - \Gamma'^{\nu}_{\mu\lambda} w_\nu$$

can be regarded as the covariant differential quotient (Erweiterung)

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[1]) Berliner Sitzungsberichte 1923 p. 32—38, 76—77, 137—140.

[2]) L.c. p. 33.

of a covariant vector, and thus the unambiguous character of this quotient would vanish. But when the second expression is used the transvection $v^\lambda w_\lambda$ of two vectors v^ν and w_λ is no more an invariant with a pseudo-parallel displacement, so that the differential quotient of the first formula occupies a well defined preferred position.

We will not consider the most general case, but the *semi-symmetrical* case in which the alternating part of the parameters has the form

$$^1/_2(\Gamma'^{\nu}_{\lambda\mu} - \Gamma'^{\nu}_{\mu\lambda}) = {}^1/_2(S_\lambda A^\nu_\mu - S_\mu A^\nu_\lambda) \quad ; \quad A^\nu_\lambda = \begin{cases} 1, \nu = \lambda \\ 0, \nu \neq \lambda, \end{cases}$$

in which S_λ is a general covariant vector [1]). It will be shown that already with this simplified supposition the above mentioned difficulty can be made to disappear.

About the *special form* of the world function Ω, nothing will be supposed, so that the resulting expressions are quite general.

2. *Deduction of the field equations.* The $\Gamma'^{\nu}_{\lambda\mu}$ of a semi-symmetrical displacement can always be divided into a symmetrical and an antisymmetrical part:

$$(1) \qquad \Gamma'^{\nu}_{\lambda\mu} = A^\nu_{\lambda\mu} + S_{[\lambda} A^\nu_{\mu]} \quad ; \quad A^\nu_{\lambda\mu} = A^\nu_{\mu\lambda} \text{ [2])}.$$

Be $R'^{\cdots\nu}_{\omega\lambda\mu}$ the curvature quantity belonging to $\Gamma'^{\nu}_{\lambda\mu}$:

$$(2) \qquad R'^{\cdots\nu}_{\omega\mu\lambda} = \frac{\partial}{\partial x^\mu}\Gamma'^{\nu}_{\lambda\omega} - \frac{\partial}{\partial x^\omega}\Gamma'^{\nu}_{\lambda\mu} - \Gamma'^{\nu}_{\varkappa\omega}\Gamma'^{\varkappa}_{\lambda\mu} + \Gamma'^{\nu}_{\varkappa\mu}\Gamma'^{\varkappa}_{\lambda\omega},$$

$R^{*\cdots\nu}_{\omega\mu\lambda}$ the curvature quantity formed in the same way with the parameters $A^\nu_{\lambda\mu}$, $R'_{\mu\lambda}$ the quantity obtained from $R'^{\cdots\nu}_{\omega\mu\lambda}$ by contracting, $\omega = \nu$:

$$(3) \qquad R'_{\mu\lambda} = \frac{\partial}{\partial x^\mu}\Gamma'^{\alpha}_{\lambda\alpha} - \frac{\partial}{\partial x^\alpha}\Gamma'^{\alpha}_{\lambda\mu} - \Gamma'^{\varkappa}_{\varkappa\alpha}\Gamma'^{\alpha}_{\lambda\mu} + \Gamma'^{\alpha}_{\varkappa\mu}\Gamma'^{\varkappa}_{\lambda\alpha}$$

and $R^*_{\mu\lambda}$ the quantity obtained in the same way from $R^{*\cdots\nu}_{\omega\mu\lambda}$, then we can easily deduce the relation

[1]) That the differences $\Gamma'^{\nu}_{\lambda\mu} - \Gamma'^{\nu}_{\mu\lambda}$ always are the components of a quantity of the third rank may be supposed as known. Cf. the author's paper in Math. Zeitschrift 13 (1922), p. 56—81, Nachtrag 15 (1922) p. 168.

[2]) In this paper the symbol $v_{[\lambda} w_{\mu]}$ means $^1/_2(v_\lambda w_\mu - v_\mu w_\lambda)$.

$$(4)\quad R'_{\mu\lambda} = R^*_{\mu\lambda} - {}^1/_2\left(\frac{\partial S_\lambda}{\partial x^\mu} - \frac{\partial S_\mu}{\partial x^\lambda}\right) + {}^1/_2(n-1)\left(\frac{\partial S_\lambda}{\partial x^\mu} - A^\nu_{\lambda\mu} S_\nu\right) - {}^1/_4(n-1) S_\lambda S_\mu =$$

$$= R^*_{\mu\lambda} - \nabla^*_{[\mu} S_{\lambda]} + {}^1/_2(n-1)\nabla^*_\mu S_\lambda - {}^1/_4(n-1) S_\lambda S_\mu,$$

in which ∇^* is the covariant differential operator belonging to $A^\nu_{\lambda\mu}$. We suppose that the determinant $R' = R'_{\lambda\mu}$ does not vanish. Hence there exists an inverse quantity $r'^{\lambda\mu}$:

$$(5)\quad R' r'^{\mu\lambda} = \frac{\partial R'}{\partial R'_{\lambda\mu}} \quad ; \quad r'^{\nu\mu} R'_{\mu\lambda} = r'^{\mu\nu} R'_{\lambda\mu} = A^\nu_\lambda .$$

When $F'_{\mu\lambda}$ and $G'_{\mu\lambda}$ are the antisymmetrical and the symmetrical part of $R'_{\mu\lambda}$:

$$(6)\quad F'_{\mu\lambda} = R'_{[\mu\lambda]} \quad ; \quad G'_{\mu\lambda} = R'_{(\mu\lambda)}\ ^1)$$

and when the word function $\mathfrak{H} = H\sqrt{-R}$ (scalar density) is a still unknown function of $G'_{\mu\lambda}$ and $F'_{\mu\lambda}$, we then have the variation equation:

$$(7)\quad \bar{d}\int \mathfrak{H}\, d\tau = \int \mathfrak{v}'^{\lambda\mu}\, \bar{d}R'_{\mu\lambda}\, d\tau = 0\ ^2),$$

in which

$$(8a)\quad \mathfrak{v}^{\prime\lambda\mu} = v^{\prime\lambda\mu}\sqrt{-R'} = (g'^{\lambda\mu} + f'^{\lambda\mu})\sqrt{-R'}$$

$$(8b)\quad g'^{\lambda\mu}\sqrt{-R'} = \frac{\partial \mathfrak{H}}{\partial G'_{\mu\lambda}} \quad ; \quad f'^{\lambda\mu}\sqrt{-R'} = \frac{\partial \mathfrak{H}}{\partial F'_{\mu\lambda}}.$$

When we substitute into (7) the value of (4), we get for $n = 4$

$$(9)\quad 0 = \int \mathfrak{v}'^{\lambda\mu}\, d\tau \left\{ \bar{d}R^*_{\mu\lambda} - {}^1/_2\bar{d}\left(\frac{\partial S_\lambda}{\partial x^\mu} - \frac{\partial S_\mu}{\partial x^\lambda}\right) + 2\bar{d}\left(\frac{\partial S_\lambda}{\partial x^\mu} - \Gamma^\nu_{\lambda\mu} S\right) - {}^3/_4\bar{d}(S_\lambda S_\mu)\right\},$$

an equation that, $R^*_{\mu\nu}$ being independent of S_λ, is equivalent with the two equations

$$(10)\quad d A^\alpha_{\varkappa\mu}\{-A^\mu_\alpha(\nabla^*_\beta v'^{\lambda\beta} - P_\beta v'^{\lambda\beta}) + \nabla_\alpha v'^{\lambda\mu} - P_\alpha v'^{\lambda\mu} - {}^3/_2 S_\alpha v'^{\lambda\mu}\} = 0$$

$$(11)\quad d S_\lambda\{\nabla^*_\mu f'^{\mu\lambda} - P_\mu f'^{\lambda\mu} - {}^3/_2(\nabla^*_\mu v'^{\lambda\mu} - P_\mu v'^{\lambda\mu}) - {}^3/_2 S_\mu g'^{\lambda\mu}\} = 0,$$

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1) In this paper $v_{(\lambda} w_{\mu)}$ means ${}^1/_2(v_\lambda w_\mu + v_\mu w_\lambda)$.

2) We use the variation symbol $\bar{d}$ in place of δ to prevent confusion with the symbol δ of the covariant differentiation.

in which P_λ is a vector depending on $R'_{\mu\lambda}$ and $r'^{\lambda\mu}$ in the following way:

$$P_\mu = {}^1/_2\, R'_{\lambda\nu}\, \nabla^*_\mu\, r'^{\nu\lambda} = -\frac{\partial \log \sqrt{-R'}}{\partial x^\mu} + A^\alpha_{\alpha\mu}. \tag{12}$$

Since $A^\nu_{\lambda\mu}$ is symmetrical in $\lambda\mu$, we get from (10):

$$\boxed{-A^{(\mu}_\alpha\, \nabla^*_\beta\, g'^{\lambda)\beta} + A^{(\mu}_\alpha\, P_\beta\, g'^{\lambda)\beta} - A^{(\mu}_\alpha\, \nabla^*_\beta\, f'^{\lambda)\beta} + A^{(\mu}_\alpha\, P_\beta\, f'^{\lambda)\beta} + \nabla^*_\alpha\, g'^{\lambda\mu} - P_\alpha\, g'^{\lambda\mu} - {}^3/_2\, S_\alpha\, g'^{\lambda\mu} = 0} \tag{I}$$

and from (11):

$$\boxed{\nabla^*_\mu\, f'^{\lambda\mu} - P_\mu\, f'^{\lambda\mu} - {}^3/_2 (\nabla^*_\mu\, r'^{\lambda\mu} - P_\mu\, r'^{\lambda\mu}) - {}^3/_2\, S_\mu\, g'^{\lambda\mu} = 0.} \tag{II}$$

For $\nabla^*_\mu f'^{\lambda\mu} - P_\mu f'^{\lambda\mu}$ we introduce the notation i'^λ. It is easily shown that

$$i'^\lambda = \nabla^*_\mu\, f'^{\lambda\mu} - P_\mu\, f'^{\lambda\mu} = \frac{1}{\sqrt{-R'}}\, \frac{\partial\, f'^{\lambda\mu} \sqrt{-R'}}{\partial\, x^\mu}. \tag{13}$$

From (*I*) follows by contracting, $\alpha = \mu$:

$$\nabla^*_\mu\, g'^{\lambda\mu} - P_\mu\, g'^{\lambda\mu} = -\, i'^\lambda - S_\mu\, g'^{\lambda\mu}. \tag{14}$$

When this value is substituted into (*I*), we get

$$\nabla^*_\alpha g'^{\lambda\mu} - P_\alpha\, g'^{\lambda\mu} = -\, {}^1/_2\, A^{(\mu}_\alpha\, i'^{\lambda)} - A^{(\mu}_\alpha\, g'^{\lambda)\beta}\, S_\beta + {}^1/_2\, S_\alpha\, g'^{\lambda\mu}. \tag{15}$$

In the supposition that also the determinant $|g'^{\lambda\mu}|$ does not vanish this equation can be simplified by the introduction of the tensor

$$g^{\lambda\mu} = \frac{\sqrt{-R'}}{\sqrt{-g}}\, g'^{\lambda\mu} \quad ; \quad g = |g^{\lambda\mu}|^{-1} \tag{16}$$

as *fundamental tensor* and the vector

$$i_\nu = \frac{\sqrt{-R'}}{\sqrt{-g}}\, i'^\nu. \tag{17}$$

Then, because of

$$P_\mu - {}^1/_2\, g_{\lambda\nu}\, \nabla^*_\mu\, g^{\lambda\nu} = -\frac{\partial}{\partial x^\mu} \log \frac{\sqrt{-R'}}{\sqrt{-g}}, \tag{18}$$

the equation (15) passes into:

$$(19)\quad \nabla^*_\alpha g^{\lambda\mu} - {}^1/_2\, g^{\lambda\mu}\, g_{\beta\gamma}\nabla_\alpha g^{\beta\gamma} = -{}^3/_8\, A^{(\mu}_\alpha i^{\lambda)} - A^{(\mu}_\alpha S^{\lambda)} + {}^3/_8\, S_\alpha g^{\lambda\mu}.$$

Transvection of this equation with $g_{\lambda\mu}$ gives:

$$(20)\quad -g_{\beta\gamma}\nabla_\alpha g^{\beta\gamma} = -{}^3/_8\, i_\alpha + 5\, S_\alpha,$$

so that we get the resulting equation:

$$(21)\quad \nabla^*_\alpha g^{\lambda\mu} = -{}^3/_8\, A^{(\mu}_\alpha i^{\lambda)} + {}^1/_8\, i_\alpha g^{\lambda\mu} - A^{(\mu}_\alpha S^{\lambda)} - S_\alpha g^{\lambda\mu}$$

and

$$(III)\quad \boxed{\nabla'_\alpha g^{\lambda\mu} = -{}^3/_8\, A^{(\mu}_\alpha i^{\lambda)} + {}^1/_8\, i_\alpha g^{\lambda\mu} - 2\, S_\alpha g^{\lambda\mu},}$$

in which ∇' is the differential operator belonging to $\Gamma'^{\nu}_{\lambda\mu}$.

From (21) we deduce:

$$(22)\quad A^{\nu}_{\lambda\mu} = \left\{ {\lambda\mu \atop \nu} \right\} - {}^1/_8\, g_{\lambda\mu} i^\nu + {}^1/_8 A^\nu_\lambda\, i_\mu + {}^1/_8 A^\nu_\mu\, i_\lambda - {}^1/_8 A^\nu_\lambda S_\mu - {}^1/_8 A^\nu_\mu S_\lambda.$$

so that, with regard to (1):

$$(23)\quad T'^{\nu}_{\lambda\mu} = \left\{ {\lambda\mu \atop \nu} \right\} - {}^1/_8\, g_{\lambda\mu} i^\nu + {}^1/_8\, A^\nu_\lambda i_\mu + {}^1/_8\, A^\nu_\mu i_\lambda - A^\nu_\lambda S_\mu.$$

Substituting (22) into (3), we obtain:

$$(24)\quad R^*_{\mu\lambda} = K_{\mu\lambda} + {}^1/_8\,(\nabla^*_\mu i_\lambda - \nabla^*_\lambda i_\mu) + {}^1/_8\, i_\mu i_\lambda - {}^1/_8(\nabla^*_\mu S_\lambda - \nabla^*_\lambda S_\mu) - \\ - {}^3/_8\, \nabla_\mu S_\lambda + {}^3/_8\, S_\mu S_\lambda,$$

in which $K_{\lambda\mu}$ is the contracted curvature quantity $K^{\cdot\cdot\cdot\nu}_{\omega\mu\lambda}$ belonging to the fundamental tensor $g_{\lambda\mu}$. By substituting (24) into (4) we obtain the field equations:

$$(IV)\quad \boxed{\begin{aligned} R'_{\mu\lambda} &= K_{\mu\lambda} + {}^1/_8\,(\nabla^*_\mu i_\lambda - \nabla^*_\lambda i_\mu) + {}^1/_8\, i_\mu i_\lambda - (\nabla^*_\mu S_\lambda - \nabla^*_\lambda S_\mu) \\ &= K_{\mu\lambda} + {}^1/_8\left(\frac{\partial i_\lambda}{\partial x^\mu} - \frac{\partial i_\mu}{\partial x^\lambda}\right) + {}^1/_8\, i_\mu i_\lambda - \left(\frac{\partial S_\lambda}{\partial x^\mu} - \frac{\partial S_\mu}{\partial x^\lambda}\right) \end{aligned}}$$

From (IV) follows for the bivector $F'_{\mu\lambda}$ of the electromagnetic field:

$$F'_{\mu\lambda} = R'_{[\mu\lambda]} = {}^1/_6 \left(\frac{\partial i_\lambda}{\partial x^\mu} - \frac{\partial i_\mu}{\partial x^\lambda} \right) - \left(\frac{\partial S_\lambda}{\partial x^\mu} - \frac{\partial S_\mu}{\partial x^\lambda} \right). \tag{25}$$

We now return to the equation (11) obtained from the variation principle. With regard to (13), (14) and (17) this equation leads to

$$i^\nu = 0. \tag{26}$$

Since i^ν has the character of a current vector, it is not allowed to consider variations of the *alternating* part of $\Gamma'^{\nu}_{\lambda\mu}$, when we wish to keep the current vector in the equations. In regions where only an electromagnetic field exists and no current, the variation principle remains valid without any restriction.

The expressions (IV) and (25) only differ from those of EINSTEIN by the terms in S_λ, hence an electromagnetic field is also possible in places with vanishing current vector i^ν. There the vector S_λ behaves as a potential vector.

We can further make the following important remarks:

1. In the field equations (IV) S_λ does not contribute to the *symmetrical* part of $R'_{\mu\lambda}$.

2. When there is no current the displacement is by (III) *conformal*, the fundamental tensor being diminished with $2\, dx^\alpha\, S_\alpha\, g^{\lambda\mu}$ when the pseudoparallel displacement is dx^ν.

3. When there is no current and no potential (23) passes into the ordinary equation of the gravitational field, in the same way as EINSTEIN's equation.

3. *The potentialvector* S_λ. It is remarkable that here the potential vector S_λ occurs as unambiguously determinated, not as a vector to which an arbitrary gradient vector may be added. This difficulty disappears when we make the supposition that the parameters which define the displacement are not the same for covariant and for contravariant vectors[1]) and thus no longer adopt the invariance of transvection. It is namely possible to alter covariant parameters independently of the transformation of the original variables by changing the *measure*[2]) of the *covariant* vectors. This change of measure

[1]) For these displacements cf. the above mentioned paper in Math. Zeitschrift **13**.

[2]) This change of measure has nothing to do with an introduction of a ds.

(27) $$\tau\, {}'w_\lambda = w_\lambda$$

in which τ is an arbitrary function, leaves the parameters of the contravariant displacement unaltered, while the covariant parameters, which we will also further denote with $\Gamma'^{\nu}_{\lambda\mu}$, will be transformed in the following way:

(28) $$'\Gamma'^{\nu}_{\lambda\mu} = \Gamma'^{\nu}_{\lambda\mu} - \frac{\partial\, lg\, \tau}{\partial x^\mu} A^\nu_\lambda .$$

Such a change of measure cannot be applied in the same easy way to contravariant vectors, the new components $\tau^{-1}\, dx^\nu$ being in general no more exact differentials. In this case we would be obliged to consider space-time as a system of non-exact differentials, and it would no more be possible to represent a point by four finite coordinates. This case has doubtlessly but little attraction so long as there are other possibilities.

When we wish to "loose" the vector S_ν in the above mentioned sense, we have only to consider the $\Gamma'^{\nu}_{\lambda\mu}$ as the parameters of the *covariant* displacement and to define the $\Gamma^{\nu}_{\lambda\mu}$, the parameters of the contravariant displacement, in the following way:

(29) $$\Gamma_{\lambda\mu} = \Gamma'^{\nu}_{\lambda\mu} + S_\mu A^\nu_\lambda = \left\{ \begin{matrix} \lambda\mu \\ \nu \end{matrix} \right\} - {}^1/_2\, g_{\lambda\mu} i^\nu + {}^1/_6\, A^\nu_\lambda\, i_\mu + {}^1/_6\, A^\nu_\mu\, i_\lambda ,$$

We then have obtained that $\Gamma^{\nu}_{\lambda\mu}$ is independent of S_λ and that, when covariant measure is changed, S_λ is transformed in the following way:

(30) $$'S_\mu = S_\mu + \frac{\partial\, lg\, \tau}{\partial x_\mu} .$$

It is very remarkable that by (23) $\Gamma'^{\nu}_{\lambda\mu}$ has just a form that leads to the desired transformation of the potential vector. If f.i. $\Gamma^{\nu}_{\lambda\mu}$ contained a term with $S_\lambda A^\nu_\mu$, it would not be possible to obtain an equation of the form (30).

Representing the covariant differential operator determined by $\Gamma^{\nu}_{\lambda\mu}$ and $\Gamma^{\nu}_{\lambda\mu}$ by ∇, (III) is changed into:

(*III'*) $$\nabla_\alpha\, g^{\lambda\mu} = -\,{}^3/_2\, A^{(\mu}_\alpha\, i^{\lambda)} + {}^1/_2\, i_\alpha\, g^{\lambda\mu}$$
$$\nabla_\alpha\, g_{\lambda\mu} = -\,{}^1/_2\, g_{\alpha\mu}\, i_\lambda - {}^1/_2\, g_{\alpha\lambda}\, i_\mu + {}^1/_2\, i_\alpha\, g_{\lambda\mu} - 2\, S_\alpha\, g_{\lambda\mu} .$$

The tensor $g_{\lambda\mu}$ is a quantity variable with transformation of covariant measure, for its components do *not* change, while the

components of a genuine quantity of second order obtain the factor τ^{-2}. When the current vanishes, this quantity has the same character as the variable fundamental tensor of WEYL's theory, and $-2\,S_\alpha$ behaves as the vector which WEYL calls φ_α.

4. *On the law of conservation of energy and momentum.* The law of conservation of energy and momentum in gravitation theory is a consequence of the identity of BIANCHI. The form of this identity is known for non-symmetrical displacements and for displacements with non-invariant transvection [1]). Hence it must be possible to deduce, starting with this identity, an equation that can be considered as an analogon of the equation that expresses the law of energy and momentum. This possibility exists already before any supposition is made relating to the special form of Hamilton's function.

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[1]) Cf. Math. Zeitschrift 1923, 17, p. 111—115; R. WEITZENBÖCK, Invariantentheorie (NOORDHOFF, Groningen 1923), p. 357.